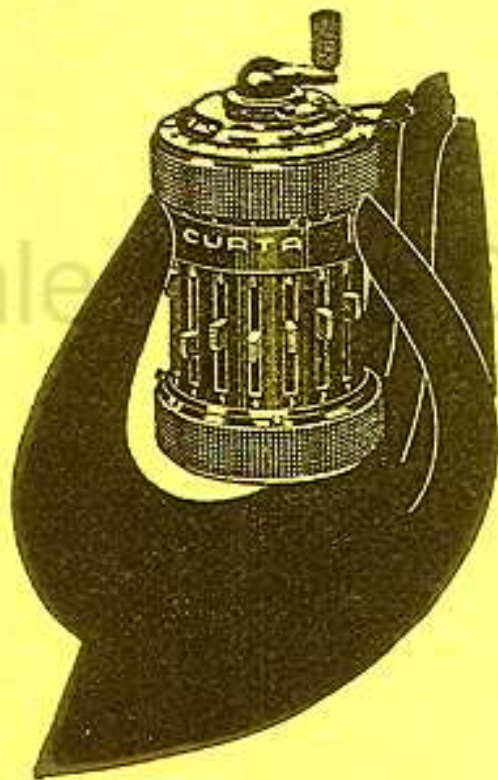


**CURTA**

**CALCULATING TECHNIQUES**



**AUTOMATIC BUSINESS MACHINES LIMITED**

**15, CROMWELL ROAD, LONDON, S.W.7.**





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# CALCULATION WITH THE CURTA

## P R E F A C E

This manual is primarily intended to supplement the Manufacturers' little handbook 'Instructions for the use of the Curta Calculating Machine' which should first be read by the Curta user. It is hoped, however, that it will prove useful to calculating machine users generally.

The operation of the machine itself, whether Model I or Model II, is so simple that many users may not at first appreciate the large variety of calculations which can be solved with the Curta. Basically, the Curta only performs repeated addition, but there are few operations of numerical analysis and virtually no commercial calculations which cannot be reduced to a process of repeated addition.

It is the purpose of this manual to show the most suitable method or 'programme' by which common commercial and scientific calculations can best be adapted for solution by the Curta. Further instruction sheets will be made available from time to time and it is hoped that readers will send us a note of any methods, not so far indicated, which they have found useful for special purposes. In this way, a really comprehensive 'library' can be built up. Every six months we will circularise users with a copy of the latest Index so that they may apply for any new instruction sheets likely to be of interest to them.

Anyone who has attempted to put instructions for calculating procedure on paper will appreciate the difficulty of explaining a technique clearly, yet concisely. Calculations which take only a few seconds to perform are liable to take as many minutes to describe, and Examples are more useful than detailed descriptions. For these reasons we have evolved an abbreviated notation for describing the process of solution which we hope will be readily understood.

We strongly recommend that the examples should be worked out on a Curta when the instructions are first read. Generally, a Model I will be sufficient unless otherwise indicated.

The various uses of the Curta will be indexed by a letter showing the general type of calculation and a serial number indicating the particular calculation described.

ABBREVIATED NOTATION

The Curta has 3 Registers, or groups of dials, which will be indicated by their initials:-

|                   |    |
|-------------------|----|
| Setting Register: | SR |
| Counter Register: | CR |
| Product Register: | PR |

A carriage position will be indicated thus:-

|                         |    |
|-------------------------|----|
| Carriage to position 3: | C3 |
|-------------------------|----|

The carriage position indicated is normally that for the 1st step of the calculation.

The position of the Reversing Lever, that is the Lever which reverses the gearing of CR, will be indicated by an R immediately after the Carriage Position when the reversing lever is down, i.e. in the Reverse Position (to count negative turns). If the lever is up, the R will be omitted.

The position of decimal markers will be indicated as follows:-

|   |    |
|---|----|
| Decimal place:  | .  |
| Decimal places where markers are also set for other purposes: | :  |
| Marker used for other purpose:                                | ,  |
| Where two markers are required, e.g. to indicate a split SR:  | ,, |

In order to indicate the position of a setting or result on the corresponding register and to avoid printing unnecessary zeros, an oblique stroke / will be printed either to the left or right of a number indicating that the dials on the left or right will read 0.

To indicate that all the dials to the left of a number will read 9, a nine followed by an oblique stroke / will be placed on the left of the number, thus 9/.



Examples:

A setting of 1.534/ indicates that 1.534 has been set on the four left hand dials of SR.

9/7,641:27 on PR of Model I indicates that the figures 999997,641:27 appear in the PR dials. One decimal marker has been set to indicate the 'thousands' and two decimal markers to indicate the decimal place.

In showing the steps of a calculation, SR will always be placed on the left, CR in the centre and PR on the right.

An asterisk will be placed on the right of the result. Thus, if 1.534 is to be multiplied by 37.2 and the result shown positively in the extreme left hand side of PR in Model I, the abbreviated description will be:-

|    |          |       |           |
|----|----------|-------|-----------|
| C6 | /1.53400 | 37.2/ | 57.0648/* |
|----|----------|-------|-----------|

This means that we start with the carriage in position 6, the reversing lever up, set 1,534 in SR dials 6 to 3, multiply by 37.2 from the left and obtain the result 57.0648 in the left hand dials of PR.

It is sometimes desirable to set a complement into the machine or to convert a number, read from PR or CR, into its complement. This can very easily be done mentally by deducting each digit from 9, except the right hand digit, which should be deducted from 10.

|             |                    |           |
|-------------|--------------------|-----------|
| Example (i) |                    | 5390724   |
|             | Set complement as  | 9/4609276 |
|             |                    | 9/400912  |
| (ii)        | Read complement as | 599088    |

It will be found with a little practice that complements can be set or read almost as easily as the numbers themselves.

There are several techniques in which the CR dials are not used and where it is not necessary to set the Reversing Lever down. In the examples, however, the figures appearing in the CR dials and the position of the Reversing Lever are generally given and the CR dials are cleared in order to show the turns of the handle which are made at each step. Of course full advantage should be taken of short cuts so that the number of turns actually made will be many less than the reading on the CR dials.



## H A N D L I N G   T H E   C U R T A

There are a few points in handling the Curta which we do not think have been made sufficiently clear in the Makers' Handbook.

1. When holding the machine in the left hand, the third and little fingers should be clear of the setting levers to avoid moving these levers by mistake during operation.
2. The carriage should not be gripped tightly by the thumb and forefinger. When the carriage has been lifted by thumb and forefinger, pressure of the forefinger should be instinctively released and the carriage rotated by the thumb only so that it must drop into the next position. During a calculation the carriage should only be stepped one place at a time, even if no turns are to be made in a position. In this way, the carriage can be moved without looking at the machine and there will be no danger of moving two steps instead of one.
3. The crank handle should be held lightly between the thumb and forefinger of right hand. This will avoid the danger of passing over the zero position by mistake.
4. Left handed operators will generally find it more convenient to hold the Curta in their right hand, since they will wish to hold their pencil in the left hand whilst operating the machine.
5. Normally, the clearing ring should be left in the right hand position, i.e. between the first dial of PR and the last dial of CR. It is possible to clear a portion of PR or CR if desired and this can be very useful for certain calculations. If the left hand side of PR is to be cleared after each calculation, it will of course be necessary to leave the clearing ring in the left hand position.
6. The use of short cut methods of multiplication and division cannot be too strongly stressed. The operator should get accustomed to short cutting when multiplying from the left or right and when using such special techniques as successive and transfer multiplication described in Section A.  
Time saved in rotating the handle is not the only advantage of short cutting in multiplication. It also minimises mistakes in counting turns of the handle as the number of turns in any one position of the carriage (other than the left hand position) need seldom be more than five.
7. When not in use, the Curta should be replaced in its rubber-sealed metal case, both to avoid accidental damage and the collection of dust and dirt on the setting levers, resulting in stiffness of these levers. If the levers should become at all stiff, the Curta should be returned to

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We give this name to multiplication of several factors, e.g.  $a \times b \times c \times \dots$ . It is, of course, possible to re-set each partial product but this wastes time and re-setting or writing down partial products may lead to errors.

Having obtained the first partial product on PR, set the next factor on SR WITH THE RIGHT HAND OPERATIVE DIGIT (digit other than zero) reduced by 1. Step the carriage so that this digit is below the left hand digit in PR and rotate the handle through the number of turns corresponding to that digit of PR. Step the carriage one turn to the left and turn the handle through the next digit and so on.

What we are really doing is to add  $ab(c - 1)$  to  $ab$ , getting  $abc$ .

Example (i)  $123 \times .345 \times 567$

|    |       |        |             |
|----|-------|--------|-------------|
| C1 | /.345 | /123   | /42.435*    |
| C5 | /566  | /42435 | /24060.645* |

Answer: 24,060.645

Example (ii)  $24,060.645 \times 192.1$ , where the first figure is a product already produced on PR.

If we have more than the CR capacity set on PR, we turn the carriage to its extreme left hand position, 6 with Model I Curta, and set the new factor with the last digit reduced by 1 immediately under the left hand digit of PR, thus,

|    |          |        |               |
|----|----------|--------|---------------|
| C6 | /192.000 | 240606 | 4622041.2645* |
|----|----------|--------|---------------|

Answer: 4,622,041.2645

Note that we set .0 under the left hand digit of PR.

Note also that we have not been able to step the carriage so as to multiply by the two right hand digits, 4 and 5, and the result will therefore not be exact. In fact, it will have an error up to 5 in the sixth place of the answer. If we want an exact answer we must either re-set the last factor on the right of SR and multiply by the remaining digits, having stepped the carriage to the left hand of these digits, or we must re-set the last partial product and multiply by the last factor in the normal manner. Thus, with the former method, we have

|    |       |     |               |
|----|-------|-----|---------------|
| C2 | /1920 | /45 | 4622049.9045* |
|----|-------|-----|---------------|

Answer: 4,622,049.9045



An objection may be made to this method of successive multiplication on the grounds that we cannot check the multiplier, unless we write down the partial product and clear CR. A method of overcoming this is to reduce, not the right hand operative digit, but the next zero on the right, by one.

Thus, taking the same calculation as before, viz  $123 \times .345 \times 567$

|    |        |        |              |          |
|----|--------|--------|--------------|----------|
| C1 | /.345  | /123   | /42.435*     | Clear CR |
| C5 | /566.9 | /42435 | /24060.6450* |          |

Notice that as we rotate the handle through the number of terms corresponding to the digit of PR immediately above '.9', this digit is reduced to zero and therefore there is no danger that we turn through the wrong number of turns. Notice also that if we wish to use the decimal markers we must move the marker in PR one place to the left in the second stage as there is then one decimal place in SR.

There are occasions when it is required to multiply a negative quantity on PR (appearing as a complement) by a positive quantity, producing the product as a complement on PR; or by a negative quantity, producing a positive amount on PR; or to multiply a positive quantity on PR by a negative quantity, producing a negative amount on PR.

A variation of the Successive Multiplication technique is used and is applicable to such problems as evaluation of Polynomials (see M 18) where some of the terms are negative.

Example (i)  $-123 \times 47$ ,

where the complement,  $9/877$  is set on PR.

Set  $/46$  on SR. Carriage to C4 so that right hand figure '6' on SR comes below the right hand '9' on PR. Make one negative turn. Carriage to C3 and proceed as for normal successive multiplication, thus:-

|    |     |       |         |
|----|-----|-------|---------|
| C4 | /46 | 9/877 | 9/4219* |
|----|-----|-------|---------|

The amount showing on PR is the complement of  $123 \times 47$ , i.e. of 5781.

Example (ii)  $-123 \times -47$ , where  $9/877$  is set on PR.

Set  $/48$  on SR and proceed as before but making one positive turn in position C4 and negative turns thereafter, thus:-

|    |   |     |       |        |
|----|---|-----|-------|--------|
| C4 | R | /48 | 9/877 | /5781* |
|----|---|-----|-------|--------|

Example (iii).  $123 \times -47$ , where /123 is set on PR.

Set /48 on SR, carriage to C3 and proceed as in normal successive multiplication but making subtractive, instead of additive, turns.

C3 R      /48                      /123                      9/4219\*

We can thus formulate the following rules.

- (1) When multiplying by a negative quantity, add '1' to last operative figure set on SR, instead of subtracting '1', and rotate the handle in subtractive position through the number of turns corresponding to the digits on PR.
- (2) If a complement is set on PR, step the carriage so that the right hand figure on SR comes below the right hand 9/ on PR, turning the handle in the opposite direction to that used for the other digits of PR.

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TRANSFER MULTIPLICATION

We give this name to a method of multiplying a figure produced in CR by a figure set on SR, without re-setting the CR figure. This method can be very useful in series of multiplications and divisions, such as

$$\frac{a \times b \times c}{d \times e}$$

This calculation is made in stages - first  $a \times b = f$  on PR, then  $f/d = g$  on CR, then  $g \times c = h$  on PR, finally  $h/e = i$  on CR.

Example:

$$\frac{123 \times 345 \times 567}{234 \times 456}$$

|    |   |      |         |             |          |
|----|---|------|---------|-------------|----------|
| C6 |   | /345 | 123/    | /42435000*  | Clear CR |
| C6 | R | /234 | 181346* | /36         | Clear PR |
| C1 | R | /567 | 000000  | /102823182* |          |
| C6 | R | /456 | 225489* | /198        |          |

Note that in the 3rd stage, when we have to multiply a figure on CR, 181346 by 567, we merely set /567 on SR and using positive turns, reduce each digit of CR, in turn, to zero. This is very quickly done, with virtually no risk of mistake, by positive turns. Since the Reversing Lever is down, the respective figures in CR will be reduced by the number of turns made. At the same time the figure set in SR is multiplied by the number of turns made.

We can, theoretically, carry on like this indefinitely, alternately multiplying and dividing. After the first stage the Reversing Lever can be left down.

Generally it is not worth while setting decimal markers, unless a number of very similar calculations have to be made, and the decimal markers can be left in position for all stages.

If the number of factors in the numerator exceed those in the denominator by more than one, we can use successive multiplication, per method A2, at one or more stages. If the factors are identical in number, we can start with a division. If the factors in the denominator exceed those in the numerator, we must either multiply some of these together or re-set the quotient at some stage, or use the method of successive division described in A3.

## S U C C E S S I V E   D I V I S I O N

A 3

Suppose we have a figure in PR at some stage of calculation, and we want to divide it by some divisor and obtain the quotient in PR, perhaps because we want to divide the quotient by another figure. We use a method which we have called 'Successive Division', because of the analogy to successive multiplication. The method is sometimes called 'Complementary Division'.

### Example:

Divide 567 (already produced on PR as, say, 56700000)  
by 4560

Set the complement of 456 on SR, preceded by a figure 9, thus

C6            /9544            124342            /124342(0048)\*

Answer:    /.124342

Note that although extra figures may be produced in PR, the quotient can only be obtained accurately to the same number of figures as the capacity of CR. Note too that the quotient is also produced in CR and this can be used as a check that the calculation has been performed accurately. Successive division can be very useful but takes a little practice. The operator should carry the two left hand figures of the divisor in his mind during the operation.

Since the quotient remains in the PR, it can be divided by a further divisor, either by subtractive division or by successive division.



In a division calculation, instead of setting the dividend on PR and proceeding by Subtractive Division, there may, in some cases, be an advantage in setting the complement of the dividend on PR and building up PR to zero with the divisor set in SR. This is particularly the case with such calculations as  $\frac{a \times b}{c}$ , carried out in one operation, when we wish to obtain the maximum capacity of which the machine is capable.

Example:

$$\frac{123 \times 456789}{234567}$$

the answer required to 6 places of decimals.

Using a Model II,

|    |   |             |         |                 |          |
|----|---|-------------|---------|-----------------|----------|
| C6 | R | /456789     | /100000 | 9/54321100000   | Clear CR |
| C6 |   | 01230234567 | /194737 | 239526509973879 |          |

Answer = 239527

We use the right hand side of SR, 234567 to build up the right hand side of PR to as near zero as possible. Meanwhile, the left hand side of SR, 0123, is multiplied automatically by the quotient of 456789/234567 and the product shown in the left hand side of PR.

Note: The positioning of the Reversing Lever and the clearing of CR are actually unnecessary since the CR dial is not used, but they are specified here to show that the handle has been given one negative turn in the 6th position of the carriage for the 1st operation and the quotient is shown on CR in the 2nd operation.

Note also that if we had made the calculation by setting 01230234567 in SR and building up to 456789 in PR, we should have had the wrong answer, 239572, because the left hand portion of the multiplicand, 456789, would have overlapped the correct answer.

ACCUMULATION OF QUOTIENTS

Quotients can be accumulated on CR by build-up division, the reversing lever being pushed up for positive terms and down for negative terms. To set the decimal marker in PR, add the maximum number of decimals in the denominators to the number of decimal places required in the answer, plus 1. Decimal markers should always be set when accumulating quotients.

Example (1)      $\frac{32.45}{1.39} + \frac{69.8}{7.465} - \frac{101.34}{11.7}$

The answer is required to three places of decimals.

Decimal markers are set before positions 3, 7 and 4 in SR, PR and CR.

|      |        |           |              |          |
|------|--------|-----------|--------------|----------|
| C6   | /1.390 | /23.3453* | /32.4499670  | Clear PR |
| C6   | /7.465 | /32.6956* | /69.7999895  | " "      |
| C6 R | 11.700 | /24.0341* | /101.3395500 |          |

Answer 24.034

When the numerators are the sum or differences of other numbers the accumulation can still be carried out in the above way, but using subtractive division, provided care is taken to correct CR as in the example below. In calculating the position for the decimal marker in PR, we must add to the maximum number of decimal places in denominators or numerators, the number of decimal places required in the answer, plus 1.

Example (2)      $\frac{13.475 - 5.75}{6.29} + \frac{17.24 + 3.92}{7.86} - \frac{18.715}{9.5}$

The answer is required to three decimal places.

Decimal markers are set before positions 3, 7 and 4 of SR, PR and CR.



|    |   |         |          |              |          |
|----|---|---------|----------|--------------|----------|
| C5 | R | /13.475 | 99/      | /13.4750000* |          |
|    | R | /5.750  | 0        | /7.7250000*  |          |
| C5 | R | /6.290  | /1.2281* | /0.0002510   | Clear PR |
| C5 | R | /17.240 | /1.2281  | /17.2400000  |          |
|    | R | /3.920  | 9/.2281  | /21.1600000  |          |

Now clear SR and make two negative turns to correct CR.

|    |   |        |         |             |          |
|----|---|--------|---------|-------------|----------|
| C5 | R | 0      | /1.2281 | /21.1600000 |          |
| C5 | R | /7.860 | /3.9202 | /00.0000940 | Clear PR |

The last quotient can be obtained by building-up division, but we keep reversing lever down because it has to be subtracted.

|    |   |        |         |             |  |
|----|---|--------|---------|-------------|--|
| C5 | R | /9.500 | /1.9502 | /18.7150000 |  |
|----|---|--------|---------|-------------|--|

Answer 1.950

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(a) Decimalizing

Sterling calculations are normally carried out by converting shillings and pence into decimals of £1, or pence into decimals of one shilling. The latter is used mainly for small amounts.

Curta Tables 1 and 2 show the decimal equivalents of pence and fractions of pence expressed as decimals of £1 and one shilling respectively.

Curta Table 5 shows shillings and pence as decimals of £1; shillings can, however, easily be expressed as decimals of £1 by dividing by 2. Thus 17/- = .85 of £1

Example (i)

Find the cost of 455 articles @  $3/6\frac{1}{2}$ d. each.

From Table 2,  $6\frac{1}{2}$  is equivalent to .54167 of 1/-

We therefore set 3.54167 on SR and multiply by 455 in the normal manner.

|    |          |      |                           |
|----|----------|------|---------------------------|
| C1 | /3.54167 | /455 | /1,611.45985 <sup>*</sup> |
|----|----------|------|---------------------------|

Mentally divide 1,611 shillings by 20 obtaining £80. 11/- .. Looking up the nearest figure to .45985 on Table 2 we obtain .4583 or  $5\frac{1}{2}$ d., so that the answer is:-

£80. 11.  $5\frac{1}{2}$ d.

The pence may alternatively be found by mentally multiplying the decimal by 12 or by memorising the first two decimal places, if much of this work is being done.

Example (ii)

Find the cost of 215 shares @ £1. 3.  $7\frac{1}{2}$ d. each.

Set the pounds on SR followed by the shillings divided by 2 with, say, 5 decimal places, thus 1.15000

From Table 1,  $7\frac{1}{2}$ d. is equivalent to .03125 of £1. We therefore add this to the previous setting, so that 1.18125 is set on SR.

|    |          |      |                         |
|----|----------|------|-------------------------|
| C1 | /1.18125 | /215 | /253.96875 <sup>*</sup> |
|----|----------|------|-------------------------|

The answer is thus £253.96875 or £253. 19.  $4\frac{1}{2}$ d.

The shillings can be found by reducing the first two decimal figures to a multiple of 5, in this case 95, and multiplying by 2. The remainder, .01875, can be looked up in Table 1, giving  $4\frac{1}{2}$ d, or we can regard the 2nd and 3rd places of decimals, in this case 18, as farthings, mentally dividing by 4 to obtain  $4\frac{1}{2}$ d. If the pence figure thus found comes to 6d. or over we deduct a farthing. Thus, for .037, which lies between  $8\frac{1}{2}$  and 9d, we obtain  $9\frac{1}{2}$ d. less  $\frac{1}{4}$ d., equals 9d.



Example (iii)

Find the cost of 5,150 articles at £3. 17. 7d.

From Table 5, we obtain .8791667 for 17/7d.

C1            3.8791667        /5,150        19,977.708505\*

Answer = £19,977. 14. 2d.

To decide the number of decimal places to be set on SR, it is advisable to add the number of digits in the multiplier to 3, if decimals of £1 are to be set, or 2 if decimals of 1/- are set. This will enable the answer to be given to the nearest farthing. Thus, in the above example, we had 4 figures in the multiplier so that we set 7 decimal places of £1.

(b) Pence Transfer

The disadvantages of decimalising sterling are that a table is required, unless a skilled operator is employed who can memorise the decimal equivalents, and, if the multiplier is large, a considerable number of decimal places have to be set. An alternative method, which is especially useful when the sum, only, of a number of sterling products is required, is to set the shillings and pence, separated by several zeros. After multiplication, the PR shows the answer in shillings and pence. The surplus pence (in excess of 11d.) can be converted into shillings by an adaptation of the method of Successive Division given in A 3.

Example (i)

Check the following Invoice in total.

|                              |                     |
|------------------------------|---------------------|
| 125 dozen @ 3/7d. per dozen  | 22. 7. 11d.         |
| 65 articles @ 13/11d. each   | 45. 4. 7d.          |
| 75 lbs. @ £1. 3. 5d. per lb. | 87. 16. 3d.         |
|                              | <u>£155. 8. 9d.</u> |

C1            /3,000007        /125        /375,000875\*        Clear CR

C1            13,000011        /65        /1220,001590\*        Clear CR

C1            23,000005        /75        /2945,001965\*        Clear CR

Now follows the transfer of the surplus pence

C3            /,999988        /163        /3108,000009\*

Answer = 3108 shillings and ninepence = £155. 8. 9d.

As the operation is one of checking only, it is not really necessary to Clear CR. The number of zeros to be left between shillings and pence depends on the quantity of items and number of entries on the invoice. Here we could have reduced the number of zeros by 2. If, at some stage of the calculation, the pence threaten to flow over to the shillings in the PR, we can always make a pence transfer and then continue with the other items on the invoice. In making the pence transfer, we set 88 under the right hand pence figures and 9s up to the shillings figure.

We can deal with fractions of pence by decimalising them, as this is easy.

Example (ii)

|                                    |                         |
|------------------------------------|-------------------------|
| 38 @ $2/7\frac{3}{4}$ d.           | 5. 0. $6\frac{1}{2}$ d. |
| 150 @ $13/2\frac{1}{2}$ d.         | 99. 1. 3d.              |
| 144 @ $5/11\frac{1}{2}$ d.         | 42. 15. 0d.             |
| $\pounds 146. 16. 9\frac{1}{2}$ d. |                         |

|    |            |      |                 |
|----|------------|------|-----------------|
| C1 | /2,0007.75 | /38  | /76,0294.50 *   |
| C1 | 13,0002.50 | /188 | /2026,0669.50*  |
| C1 | /5,0011.25 | /332 | /2746,2289.50 * |
| C3 | /,9988.00  | /522 | /2936,0009.50 * |

Answer =  $\pounds 146. 16. 9\frac{1}{2}$ d.

This method of checking invoices is very speedy and can be carried out by operators with little training. It gives a more independent check than checking each item. The same method may be used with advantage in checking stock sheets, in bills of quantity and even in adding figures in sterling, in which case the figures of pounds can also be separated from the shillings on Curta Model II.



CURTA TABLE NO. 1.

PENCE TO DECIMALS OF 1/-  
INCHES TO DECIMALS OF 1 FT.

| <u>PENCE</u> | <u>d</u> | $\frac{1}{4}d$ | $\frac{1}{2}d$ | $\frac{3}{4}d$ | <u>PENCE</u> |
|--------------|----------|----------------|----------------|----------------|--------------|
| 0            | .000     | .020 83        | .0416          | .062 50        | 0            |
| 1            | .083     | .104 16        | .1250          | .145 83        | 1            |
| 2            | .166     | .187 50        | .2083          | .229 16        | 2            |
| 3            | .250     | .270 83        | .2916          | .312 50        | 3            |
| 4            | .333     | .354 16        | .3750          | .395 83        | 4            |
| 5            | .416     | .437 50        | .4583          | .479 16        | 5            |
| 6            | .500     | .520 83        | .5416          | .562 50        | 6            |
| 7            | .583     | .604 16        | .6250          | .645 83        | 7            |
| 8            | .666     | .687 50        | .7083          | .729 16        | 8            |
| 9            | .750     | .770 83        | .7916          | .812 50        | 9            |
| 10           | .833     | .854 16        | .8750          | .895 83        | 10           |
| 11           | .916     | .937 50        | .9583          | .979 16        | 11           |

Note. The final figures '3' and '6' are to be repeated for as many decimal places as are required, with the last '6' replaced by a '7'.

Thus:-  $3\frac{1}{4}d = .2708333$  and  $6\frac{1}{4}d = .5416667$  to 7 decimal places.

CURTA TABLE No. 2.

PENCE TO DECIMALS OF £1.

| <u>PENCE</u> | <u>d</u>    | $\frac{1}{2}d$ | $\frac{3}{4}d$ | $\frac{1}{4}d$ | $\frac{3}{8}d$ | <u>PENCE.</u> |
|--------------|-------------|----------------|----------------|----------------|----------------|---------------|
| 0            | .0<br>000 0 | .0<br>010 416  | .0<br>020 833  | .0<br>031 250  | 0              |               |
| 1            | 041 6       | 052 083        | 062 500        | 072 916        | 1              |               |
| 2            | 083 3       | 093 750        | 104 166        | 114 583        | 2              |               |
| 3            | 125 0       | 135 416        | 145 833        | 156 250        | 3              |               |
| 4            | 166 6       | 177 083        | 187 500        | 197 916        | 4              |               |
| 5            | 208 3       | 218 750        | 229 166        | 239 583        | 5              |               |
| 6            | 250 0       | 260 416        | 270 833        | 281 250        | 6              |               |
| 7            | 291 6       | 302 083        | 312 500        | 322 916        | 7              |               |
| 8            | 333 3       | 343 750        | 354 166        | 364 583        | 8              |               |
| 9            | 375 0       | 385 416        | 395 833        | 406 250        | 9              |               |
| 10           | 416 6       | 427 083        | 437 500        | 447 916        | 10             |               |
| 11           | 458 3       | 468 750        | 479 166        | 489 583        | 11             |               |

Note. The final figures '3' and '6' are to be repeated for as many decimal places as are required, with the last '6' replaced by a '7'.

Thus:-  $2/3d = .1333333$  and  $3/6d = .1777777$  to 8 decimal places.



CURTA TABLE NO. 3.

QUARTERS, STONES AND POUNDS AS DECIMALS OF 1 CWT.

| lbs. | QUARTERS |      | STONE |      | 1    |      | 2    |      | 3    |   | 4 |   | 5 |   | 6 |   | 7 |   | Extra<br>Figures |
|------|----------|------|-------|------|------|------|------|------|------|---|---|---|---|---|---|---|---|---|------------------|
|      | 0        | 0    | 1     | 2    | 1    | 2    | 3    | 4    | 3    | 4 | 5 | 6 | 7 | 6 | 7 | 5 | 6 | 7 |                  |
| 0    | .000     | .125 | .250  | .375 | .500 | .625 | .750 | .875 | .000 |   |   |   |   |   |   |   |   |   | 000              |
| 1    | .008     | .133 | .258  | .383 | .508 | .633 | .758 | .883 | .008 |   |   |   |   |   |   |   |   |   | 929              |
| 2    | .017     | .142 | .267  | .392 | .517 | .642 | .767 | .892 | .017 |   |   |   |   |   |   |   |   |   | 857              |
| 3    | .026     | .151 | .276  | .401 | .526 | .651 | .776 | .901 | .026 |   |   |   |   |   |   |   |   |   | 786              |
| 4    | .035     | .160 | .285  | .410 | .535 | .660 | .785 | .910 | .035 |   |   |   |   |   |   |   |   |   | 714              |
| 5    | .044     | .169 | .294  | .419 | .544 | .669 | .794 | .919 | .044 |   |   |   |   |   |   |   |   |   | 643              |
| 6    | .053     | .178 | .303  | .428 | .553 | .678 | .803 | .928 | .053 |   |   |   |   |   |   |   |   |   | 571              |
| 7    | .062     | .187 | .312  | .437 | .562 | .687 | .812 | .937 | .062 |   |   |   |   |   |   |   |   |   | 500              |
| 8    | .071     | .196 | .321  | .446 | .571 | .696 | .821 | .946 | .071 |   |   |   |   |   |   |   |   |   | 429              |
| 9    | .080     | .205 | .330  | .455 | .580 | .705 | .830 | .955 | .080 |   |   |   |   |   |   |   |   |   | 357              |
| 10   | .089     | .214 | .339  | .464 | .589 | .714 | .839 | .964 | .089 |   |   |   |   |   |   |   |   |   | 286              |
| 11   | .098     | .223 | .348  | .473 | .598 | .723 | .848 | .973 | .098 |   |   |   |   |   |   |   |   |   | 214              |
| 12   | .107     | .232 | .357  | .482 | .607 | .732 | .857 | .982 | .107 |   |   |   |   |   |   |   |   |   | 143              |
| 13   | .116     | .241 | .366  | .491 | .616 | .741 | .866 | .991 | .116 |   |   |   |   |   |   |   |   |   | 071              |

Example:- 5 stone 6 lbs = .678571 to 6 places of decimals.

QUARTERS AND POUNDS AS DECIMALS OF 1 TON.

CURTA TABLE No. 4.

| Qrs. | 0     | 1     | 2     | 3     | Qrs. | 0     | 1     | 2     | 3     | Extra<br>Figs. |
|------|-------|-------|-------|-------|------|-------|-------|-------|-------|----------------|
| lbs. | .0    | .0    | .0    | .0    | lbs. | .0    | .0    | .0    | .0    |                |
| 0    | 0000  | 1250  | 2500  | 3750  | 14   | 0625  | 1875  | 3125  | 4375  | 00             |
| 1    | 0044* | 1294* | 2544* | 3794* | 15   | 0669* | 1919* | 3169* | 4419* | 64             |
| 2    | 0089  | 1339  | 2589  | 3839  | 16   | 0714  | 1964  | 3214  | 4464  | 29             |
| 3    | 0133* | 1383* | 2633* | 3883* | 17   | 0758* | 2008* | 3258* | 4508* | 93             |
| 4    | 0178* | 1428* | 2678* | 3928* | 18   | 0803* | 2053* | 3303* | 4553* | 57             |
| 5    | 0223  | 1473  | 2723  | 3973  | 19   | 0848  | 2098  | 3348  | 4598  | 21             |
| 6    | 0267* | 1517* | 2767* | 4017* | 20   | 0892* | 2142* | 3392* | 4642* | 86             |
| 7    | 0312* | 1562* | 2812* | 4062* | 21   | 0937* | 2187* | 3437* | 4687* | 50             |
| 8    | 0357  | 1607  | 2857  | 4107  | 22   | 0982  | 2232  | 3482  | 4732  | 14             |
| 9    | 0401* | 1651* | 2901* | 4151* | 23   | 1026* | 2276* | 3526* | 4776* | 79             |
| 10   | 0446  | 1696  | 2946  | 4196  | 24   | 1071  | 2321  | 3571  | 4821  | 43             |
| 11   | 0491  | 1741  | 2991  | 4241  | 25   | 1116  | 2366  | 3616  | 4866  | 07             |
| 12   | 0535* | 1785* | 3035* | 4285* | 26   | 1160* | 2410* | 3660* | 4910* | 71             |
| 13   | 0580  | 1830  | 3080  | 4330  | 27   | 1205  | 2455  | 3705  | 4955  | 36             |

Note. For each cwt. add .05. If only 5 decimal places are required, figures marked with an asterisk should be rounded up 1.

Example:- 4 cwt 3 qrs 9 lbs = .2415179 to 7 decimal places.  
 3 cwt 2 qrs 20 lbs = .18393 to 5 decimal places.



The Curta can be used with advantage in the Wages Office, either as the sole means of calculation, or to supplement existing calculating machines at peak periods. For this purpose it has the advantages of low capital cost and simplicity of operation enabling it to be used by comparatively untrained staff.

Example

Assuming a standard working week of 44 hours,  
calculate the wage earned in the following cases.

| <u>Employee</u> | <u>Standard Wage</u> | <u>Equivalent Number<br/>of Hours Worked</u> | <u>Wage<br/>Earned.</u> |
|-----------------|----------------------|--|-------------------------|
| A. Smith        | 6. 15. 0.            | 47½  | 7. 5. 9.                |
| W. Jones        | "                    | 49   | 7. 10. 5.               |
| H. Brown        | "                    | 58¾  | 9. 2. 7.                |
| R. Robinson     | 7. 2. 6.             | 37¼  | 6. 0. 8.                |
| S. Green        | "                    | 59   | 9. 11. 1.               |

It is best to use a Curta II.

The standard hours are set on the right of SR and not altered. The standard wage is set on the left of SR and only altered when the standard wage group alters. We build up the standard hours to the equivalent number of hours worked, on the right of PR and the wage earned is shown on the left of PR. We only clear the CR and PR dials when the standard wage changes.

|    |              |        |                      |                  |
|----|--------------|--------|----------------------|------------------|
| C5 | /6.750000044 | /10795 | /7.2866250,,47.4980* |                  |
| C3 |              | /11136 | /7.5168000,,48.9984* |                  |
| C4 |              | /13352 | /9.0126000,,58.7488* | Clear CR<br>& PR |
| C5 | /7.125000044 | /8466  | /6.0320250,,37.2504* |                  |
| C4 |              | /13409 | /9.5539125,,58.9996* |                  |

The answers, as given on the left of PR, are written down on the wages sheet, as above, usually rounded off to the nearest penny above the figure calculated.

Percentages are used in a great variety of ways in Commerce and Industry. A few of these are dealt with in "Instructions for Use of the Curta". The object of these notes is to summarise the various types of percentage calculations and to show the quickest way of obtaining the answers on the Curta.

(1)  $a\%$  on  $b = \frac{a \times b}{100}$

(2) Add  $a\%$  to  $b = b + \frac{a \times b}{100}$

(3) Rebate or Discount of  $a\%$  on  $b = b - \frac{a \times b}{100}$

(4)  $a$  expressed as  $\%$ age of  $b = \frac{a \times 100}{b}$

(5)  $a$ ,  $b$  and  $c$  expressed as  $\%$ ages of their sum,  $a + b + c$   
 $= \frac{a \times 100}{a + b + c}$ , etc.

(6) Selling price to give a percentage profit of  $a\%$  on sale for  
a cost price  $b = \frac{b}{1 - \frac{a}{100}}$

(7) Increase or Decrease expressed as a percentage of the larger or smaller figure. If  $b$  is greater than  $a$

$\%$  increase or decrease on  $a = \frac{(b - a)}{a} \times 100$

$\%$  increase or decrease on  $b = \frac{(b - a)}{b} \times 100$

(8) Successive Percentage corrections,  $d + a\% + b\% - c\%$  etc.

A percentage is normally required to a certain standard number of decimal places, so it is advisable to set the decimal markers, once and for all, before the set of calculations is commenced. This will reduce the work to the minimum.



$$(1) \quad a\% \text{ on } b = \frac{a \times b}{100}$$

We merely set  $b$ , multiply by  $a$  and mark off 2 additional decimal places on PR.

Example (i)

• Calculate a dividend of  $4\frac{1}{2}\%$  on £1,115. 10. 6. stock

|    |            |      |             |
|----|------------|------|-------------|
| C1 | /1,115.525 | /4.5 | /50.198625* |
|----|------------|------|-------------|

Answer = £50. 3.  $11\frac{3}{4}$ d.

Note that there are 2 more decimal places in PR than in SR + CR. Unless the percentage is more than 100, it is normally never necessary to set more than 3 decimal places in SR for sterling amounts.

$$(2) \quad \text{Add } a\% \text{ to } b = b + \frac{a \times b}{100}$$

In such cases, as in the extension of invoices, it is often desired to know both the percentage addition and the total. If we only have one such calculation to make we can set  $b$  and multiply first by  $\frac{a}{100}$  and then by 1, without clearing PR. If we have a number of such calculations to perform with the same percentage, we can set  $\frac{a}{100}$  on the right of SR,  $1 + \frac{a}{100}$  on the left and multiply by  $b$ , obtaining the percentage on the right of PR and the total on the left.

Example (ii)

Add  $17\frac{1}{2}\%$  to an invoice for £13. 14. 7.

|    |         |        |             |
|----|---------|--------|-------------|
| C1 | /13.729 | /17.5  | /2.402575*  |
| C4 |         | /117.5 | /16.131575* |

|          |                       |   |
|----------|-----------------------|---|
| Answer = | Net amount of invoice | £13. 14. 7.                               |
|          | Add $17\frac{1}{2}\%$ | <u>2. 8. <math>0\frac{1}{2}</math>.</u>   |
|          |                       | <u>£16. 2. <math>7\frac{1}{2}</math>.</u> |

Example (iii)

Add  $12\frac{1}{2}\%$  to a number of invoices.

Using a Curta II. First invoice for £18. 4. 9.

|    |                                |         |                     |
|----|--------------------------------|---------|---------------------|
| C1 | 1.1250000.125                  | /18.237 | 20.516625:2.279625* |
|    | Answer = Net amount of invoice |         | £18. 4. 9.          |
|    | Add $12\frac{1}{2}\%$          |         | 2. 5. 7.            |
|    |                                |         | £20. 10. 4.         |
|    |                                |         | £20. 10. 4.         |

This method is obviously impractical if the amount of the invoice is large or there are too many significant figures in the percentage. The setting will be left for the next calculation.

$$(3) \quad \text{Rebate or Discount of } a\% \text{ on } b = b - \frac{a \times b}{100}$$

Precisely the same considerations apply as for (2)

Example (iv)

Deduct a discount of  $2\frac{1}{2}\%$  from £136. 15. 4.

|    |                                  |       |                            |
|----|----------------------------------|-------|----------------------------|
| C1 | /136.766                         | /2.5  | /3.419150*                 |
| C4 |                                  | /97.5 | /133.346850*               |
|    | Answer = Gross amount of invoice |       | £136. 15. 4.               |
|    | Less $2\frac{1}{2}\%$ discount   |       | 3. 8. $4\frac{1}{2}$ .     |
|    |                                  |       | £133. 6. $11\frac{1}{2}$ . |
|    |                                  |       | £133. 6. $11\frac{1}{2}$ . |

$$(4) \quad 'a' \text{ expressed as a \%age of } b = \frac{a \times 100}{b}$$

This is simply a division sum and is best carried out by build up division.

Example (v)

A mixture containing 1.137 litres includes 39 centilitres of alcohol. What is the percentage of alcohol to 3 places of decimals.

Set 3 places of decimals in CR (for the answer). There are 3 decimal places in SR, so there will be  $3 + 3 + 2$  (for %age) = 8 in PR. Set 1.137 and build up to .39 in PR.

|    |          |          |            |
|----|----------|----------|------------|
| C5 | /1.137   | /34.301* | /.39000237 |
|    | Answer = |          | 34.301%    |



(5) a, b and c expressed as percentages of  $a + b + c$

We set decimal markers on SR, CR and PR, adding 2 for PR as we require the answers as percentages.

We set  $a + b + c$  on SR and build up in turn to a, b and c in PR, without clearing any of the dials.

Example (vi)

The cost of a batch of articles is given as follows:-

|              |              |
|--------------|--------------|
| Direct Wages | £139. 14. 0. |
| Materials    | 57. 16. 6.   |
| Overheads    | 112. 10. 0.  |
|              | <hr/>        |
|              | £310. 0. 6.  |
|              | =====        |

Compute the percentage of each on the total cost, to 2 places of decimals.

|    |          |         |              |
|----|----------|---------|--------------|
| C4 | /310.025 | /45.06* | /139.6972650 |
|    |          | /18.65* | /57.8196625  |
|    |          | /36.29* | /112.5080725 |

|          |              |         |
|----------|--------------|---------|
| Answer = | Direct Wages | 45.06%  |
|          | Materials    | 18.65%  |
|          | Overheads    | 36.29%  |
|          |              | <hr/>   |
|          |              | 100.00% |
|          |              | =====   |

The carriage position is simply chosen by eye to enable the dividend to be built up correctly. In this case 4 happens to be the best position to start the build up in each case. None of the dials are cleared during the operation. The total of the percentages should, of course, equal 100% with .01% either way.

If there is a large number of constituent parts, e.g. more than 5, it is worth while computing the reciprocal of the sums and multiplying each part by this reciprocal in turn.

Thus, in this example we might compute  $100/310.025$  by build up division to 100, obtaining .32255. Set this figure on SR and multiply in turn by the three constituent parts, 139.7, 57.825 and 112.5.

(6) Selling price to give a percentage profit  $a$  on a cost price  $b$  = 
$$\frac{b}{1 - \frac{a}{100}}$$

Example (vii)

A number of articles costing  $22/9$ ,  $25/6$ ,  $31/3$ , etc. have to be written up to their selling prices to the nearest penny to give a profit of  $22\frac{1}{2}\%$  on the selling price.

If there are a large number of articles, we compute  $\frac{100}{100 - a}$ ,

i.e.  $\frac{100}{77.5}$

Set the quotient on SR and multiply by each cost price in turn. It is better to work in decimals as if we require 2 places of decimals in the answer to give the nearest penny, and as there are at most 2 digits in the shillings, it is sufficient to have 4 decimal places in the quotient. We therefore set the carriage in position 5 for evaluating  $\frac{100}{77.5}$

|    |         |          |             |               |
|----|---------|----------|-------------|---------------|
| C5 | /77.5   | /1.2903* | /.9999825   | Clear CR & PR |
| C1 | /1.2903 | ./22.75  | /29.354325* | do            |
| C2 |         | /25.50   | /32.902650* | do            |
| C3 |         | /31.25   | /40.321875* |               |

Answer =  $29/4$ ,  $32/11$ ,  $40/4$ , etc.

(7) Increase or Decrease expressed as a percentage of the larger figure,  $b$ , or the smaller figure,  $a$ .

$$\% \text{ increase or decrease on } a = \frac{b - a}{a} \times 100 = \left(\frac{b}{a} - 1\right)100$$

$$\% \text{ increase or decrease on } b = \frac{b - a}{b} \times 100 = \left(1 - \frac{a}{b}\right)100$$



We have merely to divide  $b$  by  $a$  or  $a$  by  $b$ , either by subtractive division or by build up division. We can make use of the Reversing Lever, either to subtract one from the quotient in the first case, or to give the complement of  $\frac{a}{b}$  on CR in the second case.

Example (viii)

The turnover of a business increases from £567,450 to £786,584. Find the %age increase to two places of decimals.

We require  $\frac{b - a}{a} \times 100$ . Using subtractive division we set

786584 and make one positive turn with Reversing Lever down.

|    |   |         |         |          |                    |
|----|---|---------|---------|----------|--------------------|
| C5 | R | /786584 | 99/     | 0786584/ | To not clear dials |
| C5 | R | /567450 | /38.62* | 9/848100 |                    |

Answer = 38.62%

The above operation can also be appreciated in this way. After 567450 has been set and one subtractive turn has been made, we are left with 0 on CR and  $b - a$  on PR. We proceed to divide this by  $a$  so that we obtain  $\frac{b - a}{a}$  on CR.

Example (ix)

The turnover of a business decreases from £786,584 to £672,410. Find the %age decrease to two places of decimals.

We require  $\frac{b - a}{b} \times 100$ . Using build up division, we set

786584 and build up to  $a$  with reversing lever down.

|    |         |          |             |
|----|---------|----------|-------------|
| C5 | /786584 | 9/14.52* | /6723720032 |
|----|---------|----------|-------------|

Answer = 14.52%

The rule is therefore: Set the higher of the two figures, with the reversing lever down, and place carriage in position to give required decimal places in answer. If the change is an increase, make one positive turn, set the lower figure and reduce PR to zero. If the change is a decrease, build up to the lower figure.

(8) Successive Percentage Corrections:  $d + a\% + b\% - c\%$  etc.

Such calculations are common where prices are fluctuating too rapidly for catalogues to be amended.

Example (x)

$$£5. 14. 6 + 5\% + 12\frac{1}{2}\% - 2\frac{1}{2}\%$$

We can set 5.725 and multiply successively by 1.05, 1.125 and .975. Using Curta II and method given in A1.

|    |        |            |                 |          |
|----|--------|------------|-----------------|----------|
| C1 | /5.725 | /1.05      | /6.01125*       | Clear CR |
| C6 | /1.124 | /6.01125   | /6.76265625*    | Clear CR |
| C8 | /9.740 | /6.7626562 | /6.59358979505* |          |

$$\text{Answer} = £6. 11. 10\frac{1}{2}.$$

Note: It is not necessary to clear CR nor to write anything down except the answer. Nor is it necessary to proceed with the last step beyond 4 places on CR in this particular case.

If such successive percentages have to be applied to a number of prices, it is better to work out the equivalent percentage once and for all. Thus, if we multiply  $1.05 \times 1.125 \times .975$  we obtain 1.15171875 so that  $+ 5\% + 12\frac{1}{2}\% - 2\frac{1}{2}\%$  is equivalent to  $+ 15.172\%$ . If we add this percentage to £5. 14. 6. we get 6.593597 or £6. 11.  $10\frac{1}{2}$ . as before.



Loans may be repayable in a number of ways but a common method is by equal payments at regular intervals. Such payments may be calculated to repay the loan over a fixed period or the annuities may be a fixed proportion of the loan.

(a) Repayable over a fixed period.

Part of the annuity represents interest and part repayment of capital. A schedule is required showing the amounts outstanding at the end of each period and the interest and capital portions for the next period. The amount of the annuity can be found from an annuity table.

Example:

A loan of £750,000 on 1.1.54, carrying interest at  $3\frac{1}{2}\%$  is to be repaid by 10 equal payments at half-yearly intervals. The amount is the same as if a  $1\frac{1}{2}\%$  loan were repayable at yearly intervals. The amortisation factor found from the table is .1098753442. Multiplying this on the Curta by 75, we obtain the half yearly payment £82,406.51. We shall leave the figures in decimals.

We enter the half-yearly payment on PR and set the rate of interest on SR. We multiply this negatively by the amount of the loan with the Reversing Lever down, so that we subtract the interest for the 1st half year from the half-yearly payment, leaving the capital portion in PR, whilst CR shows the amount of the loan. The former is noted down and used to multiply the interest rate, positively, and added to PR.

In this way the PR shows the capital value of the annuity for each half year and the CR shows the outstanding amount of the loan at the beginning of the half-year. The operations on the Curta II are as follows, PR and CR being scheduled at each step. The setting remains constant at the interest rate.



C1 /1098753442 /75 /82,406.508150\* Clear CR  
and PR

This gives us the  $\frac{1}{2}$  yearly payment, £82,406.51

C1 /82406.510000 /1 /82406.510000 Clear CR

| Date of Payment |    | SR | Outstanding Loan (CR) | $\frac{1}{2}$ Yearly Amortisation (PR) | Interest (obtained later) |           |
|-----------------|----|----|-----------------------|--|---------------------------|-----------|
| 1.7.54.         | C8 | R  | /1.75                 | 750,000.00                             | 69,281.51                 | 13,125.00 |
| 1.1.55.         | C1 | R  |                       | 680,718.49                             | 70,493.93                 | 11,912.58 |
| 1.7.55.         | C7 | R  |                       | 610,224.56                             | 71,727.58                 | 10,678.93 |
| 1.1.56.         | C1 | R  |                       | 538,496.98                             | 72,982.81                 | 9,423.70  |
| 1.7.56.         | C7 | R  |                       | 465,514.17                             | 74,260.01                 | 8,146.50  |
| 1.1.57.         | C1 | R  |                       | 391,254.16                             | 75,559.56                 | 6,846.95  |
| 1.7.57.         | C7 | R  |                       | 315,694.60                             | 76,881.85                 | 5,524.66  |
| 1.1.58.         | C1 | R  |                       | 238,812.75                             | 78,227.29                 | 4,179.22  |
| 1.7.58.         | C7 | R  |                       | 160,585.46                             | 79,596.26                 | 2,810.25  |
| 1.1.59.         | C1 | R  |                       | 80,989.20                              | 80.989.20                 | 1,417.31  |
|                 |    |    |                       | <u>£750,000.00</u>                     | <u>£74,065.10</u>         |           |
|                 |    |    |                       | =====                                  | =====                     |           |

After the first 9 repayments the  $\frac{1}{2}$  yearly amortisation figure, £80,989.20, should equal the outstanding amount of the loan thus proving that the calculations have been made correctly and that the  $\frac{1}{2}$  yearly amortisation has been written down correctly. There may be a small discrepancy due to rounding off.

To prove the correct writing down of the outstanding amount of the loan, the  $\frac{1}{2}$  yearly amortisation figures are accumulated, starting with the last and checked with the loan figures, thus the total of the amortisation column should amount to the total of the loan.

The interest figures are obtained by setting the  $\frac{1}{2}$  yearly payment on PR and deducting each amortisation figure in turn, restoring PR by a positive turn each time, before setting the next amortisation figure.

If the amount of the loan is not more than 7 figures, these two operations can be performed with a single setting of each amortisation figure, using the one side of PR to accumulate the outstanding loan figures and the other to obtain the interest.

The total of the Interest column should equal the total amount of the Amortisation, i.e. £824,065.10 is less the total of the loan, £750,000, that is £74,065.10.



These notes do not attempt to go into the problems of gear train design, but merely to show the application of the Curta to one type of calculation arising therefrom.

A required ratio may be obtained in the form of a relationship between two quantities, i.e. a proper fraction, e.g.  $\frac{27}{167}$ , or a decimal fraction, e.g. .16168. It is necessary to express the decimal fraction as a proper fraction to obtain the number of teeth in each gear of the train and, even if the ratio is already expressed as a proper fraction, the denominator or numerator may not factorise and may be too large, for practical purposes, as is the denominator in this case, or may require a gear which is not available. Consequently it may be necessary to find other factors, each within the required accuracy, one of which will factorise to a convenient number of teeth.

If the accuracy required is not great, the appropriate ratios can be found from a table of ratios, known as a Brocot Table. A table to the base 100 gives the decimal equivalents of all fractions from  $\frac{99}{100}$  to  $\frac{1}{100}$  in descending decimal order. A table to any base could, of course, be constructed but would be cumbersome if an accuracy, say, of 1 in 100,000 were required. There are methods of obtaining additional accuracy from tables of base 100 but they involve the making of a number of separate calculations. Fortunately a very simple method is provided by the Curta for finding a set of fractions to any required degree of accuracy with but a single setting on the Curta.

Any fraction can be expressed as a 'Continued Fraction' and a sufficient number of terms may be taken to give any required accuracy. For instance, the fractions given above can be expressed as

$$\frac{1}{6} + \frac{1}{5} + \frac{1}{2} + \text{etc.}$$

The successive terms give equivalent proper fractions which are alternately greater and less than the required fraction, until, if the fraction is determinate, a zero remainder is obtained.



It can be shown that, between successive terms, there is no fraction with a smaller denominator, so that, if we proceed to evaluate the continued fraction until the required accuracy is obtained, we can be sure that we have not left out a fraction as accurate but with a smaller denominator. In this case the successive fractions are  $1/6$ ,  $5/31$ ,  $11/68$ , etc.

The method which we are going to describe is simply a way of finding the successive continued fractions automatically with a single setting on the Curta, and at the same time giving the appropriate accuracy check figure so that we can compare it with the 'Remainder' of the numerator and see whether we have obtained the necessary accuracy. A Curta I can be used, but the method will be a little clearer if we use a Curta II.

Example (1):

Find a gear train to give the ratio  $.16168$   
with an accuracy of  $\pm .00002$

Set decimal markers before the 5th, 8th and 10th dials of PR. Set the decimal equivalent of the required fraction on left of SR and the accuracy on the right of SR. (With a Curta I we set decimal markers before the 3rd, 6th and 8th dials. In setting the accuracy, we must leave out the first 2 zeros.) As we turn the handle, the numerator appears on the left of PR and the denominator on CR. The right side of PR shows the accuracy multiplied by the denominator which we will here call the 'accuracy check'. We must turn until the numerator is sufficiently near to a whole number to give the required accuracy, i.e. until either the Remainder given in PR dials 6 - 10 is less than the accuracy check or they are together equal to more than 1.

Proceed as follows, first making 1 turn at a time until a carry over is made to PR dial 11.



|    |                |    | <u>No. of turns<br/>at a time</u> |   |
|----|----------------|----|-----------------------------------|---|
| C1 | / .16168.00002 | /7 | /1.13,176.00014                   | 1 |

Note the denominator shown in CR and deduct the number of turns made at a time, i.e 1, giving 6.

We now proceed to make 6 turns at a time. The Remainder will decrease from .13176 and we continue until the Remainder has been reduced below zero.

|    |  |     |                 |   |
|----|--|-----|-----------------|---|
| C1 |  | /37 | /5.98,216.00074 | 6 |
|----|--|-----|-----------------|---|

Write down the denominator, 37  
and deduct the previous number 6 obtaining for the next number of turns 31, i.e. 1 turn in carriage position 1 and 3 turns in position 2. The Remainder will now increase.

Thus we proceed with the remainder alternately increasing and decreasing towards zero until either the Remainder is less than the accuracy check or the sum is greater than 1. It is clear that PR dials 9 and 10 must either show 00 or 99, and if this occurs, we check dials 6, 7 and 8 with 1, 2 and 3.

|    |  |       |                  |           |
|----|--|-------|------------------|-----------|
| C1 |  | /99   | /16.00,632.00198 | 31        |
|    |  |       |                  | <u>99</u> |
| C1 |  | /167* | /27.00,056.00334 | 68        |

We now have sufficient accuracy since 56 is less than 334 so that 27/167 will give the required accuracy, but we must go on, since 167 is a prime number.

|    |  |       |                   |            |
|----|--|-------|-------------------|------------|
| C1 |  | /235* | /37.99,480.00470  | 68         |
|    |  |       |                   | <u>235</u> |
| C1 |  | /402* | /64.99,536.00804  | 167        |
| C1 |  | /569* | /91.99,592.01138  | 167        |
| C1 |  | /736* | /118.99,648.01472 | 167        |

and so on.

We thus obtain a series of fractions, all sufficiently accurate,  $\frac{27}{167}$ ,  $\frac{65}{402}$ ,  $\frac{92}{569}$ ,  $\frac{119}{736}$ , etc.

The second,  $\frac{65}{402}$ , factorises as  $\frac{5 \times 13}{2 \times 3 \times 67}$ , so that we might use as our gear train say  $\frac{5}{6} \times \frac{13}{67}$ .

If this is not suitable, e.g. because there are too few teeth in the first pair, it is generally not desirable to multiply top and bottom by a common factor and we may require a further fraction. The denominator 569 is a prime number, as can be found from a table of factors, but  $\frac{119}{736}$  factorises into  $\frac{7 \times 17}{2^5 \times 23}$

We might therefore use  $\frac{7}{23} \times \frac{17}{32}$ , or  $\frac{14}{23} \times \frac{17}{64}$  etc.

Note that the number of turns made at a time, shown in the right hand column above, are the denominators of the successive continued fractions,  $\frac{1}{6}$ ,  $\frac{5}{31}$ ,  $\frac{11}{68}$ ,  $\frac{27}{167}$

Example (2)

Find a gear train to give the ratio .2905  
within an accuracy of  $\pm .00003$

|    |                |       |                    | <u>No. of turns<br/>at a time</u> |
|----|----------------|-------|--------------------|-----------------------------------|
| C1 | / .29050.00003 | /4    | /1.16,200.00012    | 1                                 |
|    |                | /10   | /2.90,500.00030    | <u>4</u><br>3                     |
|    |                | /31   | /9.00,550.00093    | <u>10</u><br>7                    |
|    |                | /55   | /15.97,750.00165   | <u>31</u><br>24                   |
|    |                | /179* | /51*.99,950.00537  | <u>55</u><br>31                   |
|    |                | /210* | /61*.00,500.00630  | 31                                |
|    |                | /389* | /113*.00,450.01167 | <u>210</u><br>179                 |
|    |                |       |                    | etc.                              |

We thus have the fractions  $\frac{52}{179}$ ,  $\frac{61}{210}$ ,  $\frac{113}{389}$ , etc.

As 179 is a prime number and too high, the first is no use.

The second factorises as  $\frac{61}{2 \times 3 \times 5 \times 7}$ ,

so that we might use  $\frac{61}{100} \times \frac{10}{21}$  and various other combinations.



It is desirable to write down the denominators when the Remainders change sign, deducting the previous number of turns required to be made in order to find the next number of turns required to be made at a time, as in the right hand column above, but no other figures need be written down, except the accurate fractions.

With a little practice, a series of fractions can be obtained with great rapidity. Short-cutting should be used and where a Remainder approaches zero slowly, we may turn through a multiple of the indicated number of turns; e.g. where 8 turns at a time are indicated, we might turn immediately through 4 turns in the 2nd position of the carriage (40 turns), proceeding thereafter with 8 turns in the 1st position.

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SIDES OF A TRIANGLE - PYTHAGORAS.

Draughtsmen and Designers frequently require to find the third side of a right angled triangle, the other two sides of which are known. This can of course be evaluated with a slide rule or by log. tables, or tables of squares and square roots, but greater speed with less effort and less danger of error can be achieved with the Curta.

The operation consists of finding  $C = \sqrt{a^2 \pm b^2}$ . We set 'a' on SR and square. We now set 'b' on SR and square with either positive or negative turns of the handle. This gives us the radicand,  $a^2 \pm b^2$ , on PR. We find the square root by one of the methods given in Section M 1. If we already know the approximate value of C, Sabielny's method (3) is probably the quickest, otherwise method (5), using Table 101, will probably be found the simplest.

Example (i).

Find the third side of the triangle given below



|      |         |         |              |          |
|------|---------|---------|--------------|----------|
| C1   | /11.392 | /11.392 | /129.777664* | Clear CR |
| C1 R | /5.975  | /5.975  | /94.077039*  | Clear CR |

Using Table 101, set the nearest figure for  $R^2$

|    |            |    |             |
|----|------------|----|-------------|
| C1 | /94.259600 | /1 | 188.336639* |
|----|------------|----|-------------|

Multiply PR by 575

|    |         |        |               |
|----|---------|--------|---------------|
| C6 | 0.0514/ | 188337 | 9.6993554639* |
|----|---------|--------|---------------|

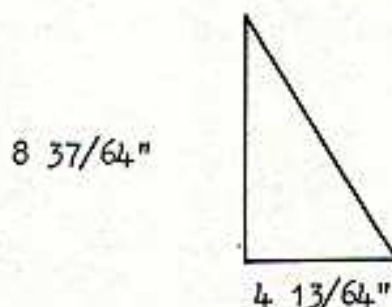
Answer 9.699"



When the measurements are expressed in fractions of an inch, e.g.  $8 \frac{37}{64}$ " , we merely convert them to 64ths, thus, multiply 8 mentally by 64 and set on SR, then add 37 by changing the setting levers.

Example (ii).

Find the third side of the triangle given below



C1            /549                            /549            /301401\*    Clear CR

C1            /269                            /269            /373762\*    Clear CR

Find square root by table 101.

C1            /372711                            /1            /746473\*    Clear CR

C6            /818                            /746473    /611.361 387\*    Clear CR

Now divide by 64.

C1            64/                            /9\*            /35361387

Answer  $9 \frac{35}{64}$ "

HEAT INSULATION

There are several calculations involved in Heat Insulation, problems which can be solved speedily on the Curta, Model II, using a method similar to those set out in A.5 for the accumulation of quotients. For instance, the heat loss,  $q$ , in BTU's per foot run of pipe, ignoring resistance of the steel pipe, is given by

$$q = \frac{\bar{u} (t_1 - t_2)}{\frac{1}{2K_1} \log_e \frac{d_1}{d} + \frac{1}{2K_2} \log_e \frac{d_2}{d_1} + \dots + \frac{1}{2K_n} \log_e \frac{D}{d_{n-1}} + \frac{1}{\mathbb{C} \times D}}$$

Where  $t_1$  and  $t_2$  are the temperatures of steam and air.

$K_1, K_2$  etc. are the thermal conductivities of layers 1, 2 etc.

$d, d_1, d_2$  etc. are the diameters of the pipe and of layers 1, 2 etc. and  $D$  is the overall diameter.

$\mathbb{C}$  is the film conductance between insulation surface and outside air.

$K_1, K_2$  etc. can be obtained from a graph which should be calibrated in terms of  $2K_1$  etc.

$\mathbb{C}$  can be obtained from a graph or tables which should be calibrated or tabulated in terms of  $\frac{1}{\mathbb{C}}$

Example :  $t_1 = 650^\circ\text{F}, t_2 = 70^\circ\text{F}.$

Insulation of  $6\frac{1}{2}$ " outside diameter pipe is  $\frac{1}{2}$ " H.T.R.,

2" Mag and  $\frac{1}{2}$ " H.S., so that  $d = 6.5, d_1 = 7.5,$

$d_2 = 11.5, d = 12.5,$  and  $2K_1 = .096, 2K_2 = .082$

and  $2K_3 = .150.$

$$\frac{1}{\mathbb{C}} = 6.67$$



Thus the amount to be computed is :

$$\frac{580 \bar{\pi}}{\frac{1}{.096} \log_e \frac{7.5}{6.5} + \frac{1}{.082} \log_e \frac{11.5}{7.5} + \frac{1}{.150} \log_e \frac{12.5}{11.5} + \frac{6.67}{12.5}}$$

We set  $\log_e 6.5$  on left of SR with carriage in position 1 and turn one subtractive turn. Then set  $\log_e 7.5$  and one positive turn bringing  $\log_e \frac{d_1}{d}$  on to PR. We set  $2K_1$ , i.e. .096. on right of SR and divide by subtraction with reversing lever down bringing quotient on CR. In this way we sum the quotient of the K terms, then set reversing lever up and add on the last quotient by setting D on SR and building up to 6.667. Finally we reset the accumulated quotients on right of SR, with  $t_1 - t_2$ , i.e. 580, set on the left and build up to  $\bar{\pi}$ , obtaining the answer on the left of PR, thus :

|    |   |               |           |               |          |
|----|---|---------------|-----------|---------------|----------|
| C1 | R | 01.8718/      | /1        | 9/8128200000* |          |
| C1 | R | 02.0149/      | /0        | /143100000*   |          |
| C7 | R | 02.0149//.096 | 01.491/*  | ....9/64000   | Clear PR |
| C1 | R | 02.0149/      | 01.491//1 | 9/7985100000* |          |
| C1 | R | 02.4424/      | 01.491/   | /427500000*   |          |
| C7 | R | 02.4424//.082 | 06.704/*  | .../34000     | Clear PR |
| C1 | R | 02.4424/      | 06.704//1 | 9/7557600000* |          |
| C1 | R | 02.5257/      | 06.704/   | /83300000*    |          |
| C6 | R | 02.5257/.150  | 07.259/*  | ..../50000    | Clear PR |
| C6 |   | /12.500       | 07.792*   | /6.662500000  |          |

Reset CR on right of SR, clear CR and PR, set 580 on left of SR and build up to  $\bar{\pi}$  (3.1416) on right of PR

|    |  |            |        |                 |  |
|----|--|------------|--------|-----------------|--|
| C4 |  | 580//7.792 | /.4032 | 233856031417344 |  |
|----|--|------------|--------|-----------------|--|

Answer 234 BTU's.

In this method, each logarithm has only to be set once and no figures need be written down except the answer. To check the calculation, it should be performed again in the reverse order. The right hand quotient will be obtained by build-up division, then the reversing lever will be pushed down,  $\log_e D$  set with one positive turn and  $\log_e d_{n-1}$  subtracted, the difference divided by  $2K_n$  and so on for the other terms.

Finally we have the accumulation of the quotients on CR. Set the answer, already obtained, on SR and bring CR to zero by positive turns. Divide by 580 and check that we have  $\pi$  on CR.

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There are various ways of finding square roots on the Curta, either with or without some other aid such as tables or a slide rule. Each method has merits according to circumstances.

(1) **Toplers Method** - Using the arithmetic series  $1 + 3 + 5 + 7 + \dots$ . This method is described in 'Instructions for the Use of the Curta'. It is comparatively slow but is simple and requires no outside aids. We give here one example in our abbreviated notation.

Example:

$$457.315^{\frac{1}{2}}$$

In all methods we divide the Radicand into groups of 2 digits from the decimal point. We build up to the first group, 4, by setting 1 and then 3 on SR and making one turn for each. We then increase SR by 1 and move the carriage down one place and commence building by the series again. Thus, with Model I:-

|    |            |           |  |
|----|------------|-----------|--|
| C6 | 001/       | 1/ *      | 1/   |
|    | 003/       | 2/ *      | 4/   |
| C5 | 0041/      | 21/ *     | 441/   |
| C4 | 00421/     | 21.1/ *   | 445.21/  |
|    | 0042.3/    | 21.2/ *   | 449.44/  |
|    | 0042.5/    | 21.3/ *   | 453.69/  |
| C3 | 0042.61/   | 21.31/ *  | 454.1161/  |
|    | .....      | .....     | ..... and so on,<br>adding 2 to<br>SR at each<br>step. |
|    | 0042.75/   | 21.38/ *  | 457.1044/  |
| C2 | 0042.761/  | 21.381/ * | 457.147161/  |
|    | .....      | .....     | .....  |
|    | 0042767/   | 21.384/ * | 457.275456/  |
| C1 | 0042.7681/ | 21.3841 * | 457.27973281   |
|    | .....      | .....     | .....  |
|    | 0042.7697  | 21.3849 * | 457.31394601   |

Thus the answer to six significant figures is 21.3849.

Notice that if we add a further 1 to SR, SR is twice CR. This proves that we have made the calculation correctly, and should always be checked.

After the first 4 digits, 2138, i.e. half the significant figures required plus 1, have been obtained, it is no longer necessary to proceed with the series 1 + 3 + 5 etc., as the SR is sufficiently accurate. We may proceed simply to build up the PR as nearly as possible to the Radicand, moving the carriage as necessary, thus:

|                  |          |            |             |
|------------------|----------|------------|-------------|
| C2               | 0042.76/ | 21.384/ *  | 457.27544/  |
| C1               |          | 21.3849/ * | 457.313924/ |
| Answer = 21.3849 |          |            |             |

## (2) Herrmann's Method

This is also given in 'Instructions for Use of the Curta'. If R is an approximate root of  $N^2$ , we may put  $N = R + d$  where d is small. Hence  $N^2 = R^2 + 2Rd + d^2$  or ignoring  $d^2$  and substituting  $N - R$  for d, we get  $N = (N^2 + R^2)/2R$

We may also express this as  $N = R + \frac{N^2 - R^2}{2R}$

$$\text{or as } N = \frac{1}{2} \left( \frac{N^2}{R} + R \right)$$

In each case N gives us a second order approximation, accurate to twice as many figures as R.

Herrmann's Method uses the second of these expressions.

If we set R on SR, square it, obtaining  $R^2$  in PR and R in CR, and then set 2R on SR, building up (or reducing) PR to  $N^2$  (the Radicand), we shall have  $R + \frac{N^2 - R^2}{2R}$  on CR.

### Example:

$$457.315^{\frac{1}{2}}$$

Suppose we find the root of 457, on a slide rule, to be 21.4

|                  |         |            |  |
|------------------|---------|------------|--|
| C6               | 0021.4/ | 21.4/      | 457.96/ *                                    |
| C3               | 0042.8/ | 21.3849/ * | 457.31372/ starting with a subtractive turn. |
| Answer = 21.3849 |         |            |  |

In the same way we could find 12 significant figures by squaring 21.3849, obtaining 457.31394801, set 42.7698 and build up the 6 right hand digits to 500000.

On Curta I it is necessary to clear PR and re-set 394801/.



After the first 4 digits, 2138, i.e. half the significant figures required plus 1, have been obtained, it is no longer necessary to proceed with the series 1 + 3 + 5 etc., as the SR is sufficiently accurate. We may proceed simply to build up the PR as nearly as possible to the Radicand, moving the carriage as necessary, thus:

|                  |          |            |             |
|------------------|----------|------------|-------------|
| C2               | 0042.76/ | 21.384/ *  | 457.27544/  |
| C1               |          | 21.3849/ * | 457.313924/ |
| Answer = 21.3849 |          |            |             |

(2) Herrmann's Method

This is also given in 'Instructions for Use of the Curta'. If R is an approximate root of  $N^2$ , we may put  $N = R + d$  where d is small. Hence  $N^2 = R^2 + 2Rd + d^2$  or ignoring  $d^2$  and substituting  $N - R$  for d, we get  $N = (N^2 + R^2)/2R$

We may also express this as  $N = R + \frac{N^2 - R^2}{2R}$

$$\text{or as } N = \frac{1}{2} \left( \frac{N^2}{R} + R \right)$$

In each case N gives us a second order approximation, accurate to twice as many figures as R.

Herrmann's Method uses the second of these expressions.

If we set R on SR, square it, obtaining  $R^2$  in PR and R in CR, and then set 2R on SR, building up (or reducing) PR to  $N^2$  (the Radicand), we shall have  $R + \frac{N^2 - R^2}{2R}$  on CR.

Example:

$$457.315^{\frac{1}{2}}$$

Suppose we find the root of 457, on a slide rule, to be 21.4

|                  |         |            |  |
|------------------|---------|------------|--|
| C6               | 0021.4/ | 21.4/      | 457.96/ *                                    |
| C3               | 0042.8/ | 21.3849/ * | 457.31372/ starting with a subtractive turn. |
| Answer = 21.3849 |         |            |  |

In the same way we could find 12 significant figures by squaring 21.3849, obtaining 457.31394801, set 42.7698 and build up the 6 right hand digits to 500000.

On Curta I it is necessary to clear PR and re-set 394801/.

With Curta II the steps are

|    |          |               |                       |
|----|----------|---------------|-----------------------|
| C8 | 21.3849/ | 21.3849/      | .31394801/ * Clear CR |
| C7 | /42.7698 | /2459656*     | .314999999951888      |
|    | Answer = | 21.3849245966 |                       |

This method is quicker than Method (1), though perhaps it is rather easier to make a mistake.

### (3) Sabielny's Method

This uses the expression  $N = \frac{1}{2} \left( \frac{N^2}{R} + R \right)$

A root R is guessed, or found on a slide rule or from tables, and this is divided into  $N^2$ . The mean of the quotient and R gives the second order approximation.

Example:

$$457.315^{\frac{1}{2}}$$

Obtain 214 as an approximate root, as in Method (1)

|    |          |         |           |
|----|----------|---------|-----------|
| C6 | /214000  | 213699* | 45731586/ |
|    | Answer = | 21.3849 |           |

This is slightly quicker than Herrmann's Method but needs perhaps a little more mental effort.

### (4) Classical Method

This is the method normally taught in elementary school arithmetic. The method is speedy and no outside aids are required, but it needs rather more mental effort than the foregoing methods.

We set the radicand on PR, set the approximate root of the left hand group, or the nearest figure below it, say x, and subtract it x times. We add x, move the carriage to the next position and set a figure y in the next column so that if we subtract y times, we shall not reduce PR below zero. Then we add y and carry on as before.

Example:

$$457.315^{\frac{1}{2}}$$

|    |          |   |          |           |          |
|----|----------|---|----------|-----------|----------|
| C6 | /457.315 |   | 1/       | 457.315/  | Clear CR |
|    | /20.0000 | R | 2/ *     | 057.315/  |          |
| C5 | /41.0000 | R | 21/ *    | 016.315/  |          |
| C4 | /42.3000 | R | 21.3/ *  | 003.625/  |          |
| C3 | /42.6800 | R | 21.38/ * | 000.2106/ |          |



As in Method (1), the setting on SR is sufficiently accurate and we need merely reduce PR to zero. Notice that if we add the last setting, 8, the SR will be twice CR, as in Method (1).

C2            /42.7600    R        21.3849 \*            /107600  
                        Answer    =    21.3849

(5) Using Curta Table 101

This method uses the expression  $N = \frac{N^2 + R^2}{2R}$

Table 101 gives a column for  $R^2$  and one for  $\frac{1}{2R}$ , the latter being spaced at intervals calculated to give 5 figure accuracy to the answer.

We set the radicand on PR, add the nearest figure to the radicand, taken from table 101, thus obtaining  $N^2 + R^2$  on PR,

and multiply this by the corresponding value of  $\frac{1}{2R}$  by the method of successive multiplication - See A 1.

Example:

457.315 <sup>$\frac{1}{2}$</sup>

|    |          |         |                      |
|----|----------|---------|----------------------|
| C1 | /457.315 | /1      | /457.315             |
|    | /456.571 | /2      | /913.886    Clear CR |
| C6 | /.0233   | 913.886 | /21.3849324 *        |

Answer    =    21.385

This method is very rapid if a 5 figure root is sufficient and requires little mental effort. In practise it is unnecessary to clear CR.

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| $R^2$     | $1/2R$ | $R^2$     | $1/2R$ | $R^2$     | $1/2R$ |
|-----------|--------|-----------|--------|-----------|--------|
| 1.00 00 0 | 500    | 3.50 68 5 | 267    | 8.45 05 1 | 172    |
| 1.03 70 0 | 491    | 3.58 70 1 | 264    | 8.65 05 2 | 170    |
| 1.07 60 8 | 482    | 3.66 99 4 | 261    | 8.75 31 9 | 169    |
| 1.11 74 2 | 473    | 3.75 57 8 | 258    | 8.85 77 1 | 168    |
| 1.16 11 9 | 464    | 3.84 46 8 | 255    | 8.96 41 1 | 167    |
| 1.20 22 9 | 456    | 3.93 67 6 | 252    | 9.07 24 3 | 166    |
| 1.24 56 1 | 448    | 4.03 21 9 | 249    | 9.18 27 4 | 165    |
| 1.29 13 2 | 440    | 4.13 11 4 | 246    | 9.29 50 6 | 164    |
| 1.33 95 9 | 432    | 4.23 37 7 | 243    | 9.40 94 6 | 163    |
| 1.38 40 8 | 425    | 4.34 02 8 | 240    | 9.52 59 9 | 162    |
| 1.43 08 3 | 418    | 4.45 08 5 | 237    | 9.64 46 9 | 161    |
| 1.47 99 8 | 411    | 4.56 57 1 | 234    | 9.76 56 3 | 160    |
| 1.53 17 1 | 404    | 4.68 50 7 | 231    | 9.88 88 5 | 159    |
| 1.58 62 0 | 397    | 4.80 91 7 | 228    | 10.01 44  | 158    |
| 1.64 36 5 | 390    | 4.93 82 7 | 225    | 10.14 24  | 157    |
| 1.69 54 2 | 384    | 5.07 26 4 | 222    | 10.27 29  | 156    |
| 1.74 96 7 | 378    | 5.16 52 9 | 220    | 10.40 58  | 155    |
| 1.80 65 7 | 372    | 5.26 05 0 | 218    | 10.54 14  | 154    |
| 1.86 62 8 | 366    | 5.35 83 7 | 216    | 10.67 97  | 153    |
| 1.92 90 1 | 360    | 5.45 89 9 | 214    | 10.82 06  | 152    |
| 1.99 49 6 | 354    | 5.56 24 8 | 212    | 10.96 44  | 151    |
| 2.05 25 3 | 349    | 5.66 89 3 | 210    | 11.11 11  | 150    |
| 2.11 26 3 | 344    | 5.77 84 8 | 208    | 11.26 07  | 149    |
| 2.17 54 1 | 339    | 5.89 12 2 | 206    | 11.41 34  | 148    |
| 2.24 10 3 | 334    | 6.00 73 0 | 204    | 11.56 93  | 147    |
| 2.30 96 6 | 329    | 6.12 68 5 | 202    | 11.72 83  | 146    |
| 2.38 15 0 | 324    | 6.25 00 0 | 200    | 11.89 06  | 145    |
| 2.45 67 4 | 319    | 6.37 69 0 | 198    | 12.05 63  | 144    |
| 2.53 56 0 | 314    | 6.50 77 0 | 196    | 12.22 55  | 143    |
| 2.60 14 6 | 310    | 6.64 25 8 | 194    | 12.39 83  | 142    |
| 2.66 99 1 | 306    | 6.78 16 8 | 192    | 12.57 48  | 141    |
| 2.74 11 1 | 302    | 6.92 52 1 | 190    | 12.75 51  | 140    |
| 2.81 51 9 | 298    | 7.07 33 4 | 188    | 12.93 93  | 139    |
| 2.89 23 1 | 294    | 7.22 62 7 | 186    | 13.12 75  | 138    |
| 2.97 26 5 | 290    | 7.38 42 2 | 184    | 13.31 98  | 137    |
| 3.05 63 8 | 286    | 7.54 74 0 | 182    | 13.51 64  | 136    |
| 3.14 37 1 | 282    | 7.71 60 5 | 180    | 13.71 74  | 135    |
| 3.23 48 2 | 278    | 7.89 04 2 | 178    | 13.92 29  | 134    |
| 3.32 99 6 | 274    | 8.07 07 6 | 176    | 14.13 31  | 133    |
| 3.42 93 6 | 270    | 8.25 73 7 | 174    | 14.34 80  | 132    |



| $R^2$    | 1/2R | $R^2$    | 1/2R | $R^2$     | 1/2R |
|----------|------|----------|------|-----------|------|
| 14.56 79 | 131  | 31.00 18 | 898  | 60.27 93  | 644  |
| 14.79 29 | 130  | 31.49 09 | 891  | 61.03 52  | 640  |
| 15.02 31 | 129  | 31.99 16 | 884  | 61.80 53  | 636  |
| 15.25 88 | 128  | 32.50 43 | 877  | 62.59 01  | 632  |
| 15.50 00 | 127  | 33.02 95 | 870  | 63.39 00  | 628  |
| 15.74 70 | 126  | 33.56 75 | 863  | 64.20 53  | 624  |
| 16.00 00 | 125  | 34.11 87 | 856  | 65.03 64  | 620  |
| 16.25 91 | 124  | 34.68 36 | 849  | 65.88 38  | 616  |
| 16.52 46 | 123  | 35.17 91 | 843  | 66.74 78  | 612  |
| 16.79 66 | 122  | 35.68 53 | 837  | 67.62 90  | 608  |
| 17.07 53 | 121  | 36.20 24 | 831  | 68.52 77  | 604  |
| 17.36 11 | 120  | 36.73 09 | 825  | 69.44 44  | 600  |
| 17.65 41 | 119  | 37.27 11 | 819  | 70.37 97  | 596  |
| 17.95 46 | 118  | 37.82 33 | 813  | 71.33 40  | 592  |
| 18.26 28 | 117  | 38.38 78 | 807  | 72.30 78  | 588  |
| 18.57 91 | 116  | 38.96 50 | 801  | 73.30 17  | 584  |
| 18.90 36 | 115  | 39.55 54 | 795  | 74.06 07  | 581  |
| 19.23 67 | 114  | 40.15 93 | 789  | 74.83 15  | 578  |
| 19.57 87 | 113  | 40.77 71 | 783  | 75.61 44  | 575  |
| 19.92 99 | 112  | 41.40 93 | 777  | 76.40 96  | 572  |
| 20.29 06 | 111  | 42.05 63 | 771  | 77.21 75  | 569  |
| 20.66 12 | 110  | 42.71 86 | 765  | 78.03 82  | 566  |
| 21.04 20 | 109  | 43.28 25 | 760  | 78.87 21  | 563  |
| 21.43 35 | 108  | 43.85 77 | 755  | 79.71 94  | 560  |
| 21.83 60 | 107  | 45.65 38 | 740  | 80.58 04  | 557  |
| 22.20 80 | 1061 | 46.27 70 | 735  | 81.45 55  | 554  |
| 22.58 96 | 1052 | 46.91 31 | 730  | 82.34 49  | 551  |
| 22.98 11 | 1043 | 47.56 24 | 725  | 83.24 90  | 548  |
| 23.38 29 | 1034 | 48.22 53 | 720  | 84.16 80  | 545  |
| 23.79 54 | 1025 | 48.90 21 | 715  | 85.10 23  | 542  |
| 24.21 88 | 1016 | 49.59 33 | 710  | 86.05 23  | 539  |
| 24.65 36 | 1007 | 50.29 93 | 705  | 87.01 83  | 536  |
| 25.10 03 | 998  | 51.02 04 | 700  | 88.00 06  | 533  |
| 25.50 76 | 990  | 51.75 72 | 695  | 88.99 96  | 530  |
| 25.92 49 | 982  | 52.51 00 | 690  | 90.01 58  | 527  |
| 26.35 25 | 974  | 53.27 93 | 685  | 91.04 95  | 524  |
| 26.79 08 | 966  | 54.06 57 | 680  | 92.10 10  | 521  |
| 27.24 01 | 958  | 54.70 75 | 676  | 93.17 09  | 518  |
| 27.70 08 | 950  | 55.36 07 | 672  | 94.25 96  | 515  |
| 28.17 33 | 942  | 56.02 57 | 668  | 95.36 74  | 512  |
| 28.65 80 | 934  | 56.70 27 | 664  | 96.49 49  | 509  |
| 29.15 53 | 926  | 57.39 21 | 660  | 97.64 25  | 506  |
| 29.60 12 | 919  | 58.09 41 | 656  | 98.81 07  | 503  |
| 30.05 73 | 912  | 58.80 91 | 652  | 99.20 48  | 502  |
| 30.52 41 | 905  | 59.53 74 | 648  | 100.00 00 | 500  |



The method of finding cube roots on the Curta is very similar to Method (5) for square roots. We use Curta Table 102.

If  $R$  is an approximate cube root of  $N^3$ , we may put  $N = R + d$ , where  $d$  is small. Thus  $N^3 = R^3 + 3R^2d + 3Rd^2 + d^3$  and if we ignore  $d^2$  &  $d^3$  and substitute  $N - R$  for  $d$  we get  $N = \frac{N^3 + 2R^3}{3R^2}$  as a second order approximation for  $N$ .

Table 102 gives a column for  $N^3$  and one for  $\frac{1}{3R^2}$ , the latter being spaced at intervals calculated to give 5 figure accuracy to the answer. The Table is arranged in groups of 3 figures from the decimal point.

Example:

|    |                         |         |             |          |
|----|-------------------------|---------|-------------|----------|
|    | $457.315^{\frac{1}{3}}$ |         |             |          |
| C1 | /457.315                | /1      | /457.315    |          |
|    | /456787                 | /3      | /1370.889   | Clear CR |
| C6 | /005610                 | 1370.89 | /7.70440179 |          |

Answer = 7.7044



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| $R^3$    | $1/3R^2$ | $R^3$    | $1/3R^2$ | $R^3$    | $1/3R^2$ |
|----------|----------|----------|----------|----------|----------|
| 1.001 50 | (.)333   | 4.407 42 | 124      | 15.294 0 | 541      |
| 1.043 52 | 324      | 4.516 25 | 122      | 15.683 8 | 532      |
| 1.083 40 | 316      | 4.629 63 | 120      | 16.090 4 | 523      |
| 1.125 88 | 308      | 4.747 83 | 118      | 16.514 8 | 514      |
| 1.171 22 | 300      | 4.871 14 | 116      | 16.908 0 | 506      |
| 1.219 67 | 292      | 4.999 89 | 114      | 17.317 1 | 498      |
| 1.264 88 | 285      | 5.134 42 | 112      | 17.742 9 | 490      |
| 1.312 96 | 278      | 5.275 08 | 110      | 18.186 4 | 482      |
| 1.364 16 | 271      | 5.422 28 | 108      | 18.648 8 | 474      |
| 1.418 77 | 264      | 5.576 47 | 106      | 19.131 1 | 466      |
| 1.468 55 | 258      | 5.738 10 | 104      | 19.570 4 | 459      |
| 1.521 31 | 252      | 5.907 69 | 102      | 20.026 7 | 452      |
| 1.577 30 | 246      | 6.085 80 | 100      | 20.501 1 | 445      |
| 1.636 82 | 240      | 6.273 05 | (.0)980  | 20.994 6 | 438      |
| 1.700 18 | 234      | 6.470 10 | 960      | 21.508 1 | 431      |
| 1.756 16 | 229      | 6.667 05 | 941      | 22.042 9 | 424      |
| 1.815 28 | 224      | 6.874 19 | 922      | 22.600 3 | 417      |
| 1.877 81 | 219      | 7.080 52 | 904      | 23.097 0 | 411      |
| 1.944 00 | 214      | 7.297 39 | 886      | 23.612 1 | 405      |
| 2.014 18 | 209      | 7.512 57 | 869      | 24.146 7 | 399      |
| 2.088 68 | 204      | 7.738 53 | 852      | 24.701 8 | 393      |
| 2.151 66 | 200      | 7.961 75 | 836      | 25.278 5 | 387      |
| 2.217 86 | 196      | 8.195 91 | 820      | 25.878 0 | 381      |
| 2.287 53 | 192      | 8.441 78 | 804      | 26.501 5 | 375      |
| 2.360 92 | 188      | 8.683 65 | 789      | 27.150 5 | 369      |
| 2.438 32 | 184      | 8.937 30 | 774      | 27.711 9 | 364      |
| 2.520 05 | 180      | 9.203 55 | 759      | 28.292 8 | 359      |
| 2.606 45 | 176      | 9.464 20 | 745      | 28.894 4 | 354      |
| 2.697 90 | 172      | 9.737 38 | 731      | 29.517 6 | 349      |
| 2.794 82 | 168      | 10.024 0 | 717      | 30.163 4 | 344      |
| 2.871 39 | 165      | 10.302 9 | 704      | 30.833 2 | 339      |
| 2.951 52 | 162      | 10.595 0 | 691      | 31.528 1 | 334      |
| 3.035 45 | 159      | 10.901 2 | 678      | 32.249 6 | 329      |
| 3.123 43 | 156      | 11.197 1 | 666      | 32.999 0 | 324      |
| 3.215 74 | 153      | 11.506 7 | 654      | 33.777 8 | 319      |
| 3.312 69 | 150      | 11.830 8 | 642      | 34.423 3 | 315      |
| 3.414 62 | 147      | 12.141 5 | 631      | 35.089 5 | 311      |
| 3.521 88 | 144      | 12.466 1 | 620      | 35.777 5 | 307      |
| 3.634 87 | 141      | 12.805 4 | 609      | 36.488 3 | 303      |
| 3.754 04 | 138      | 13.160 3 | 598      | 37.223 0 | 299      |
| 3.879 87 | 135      | 13.497 5 | 588      | 37.982 6 | 295      |
| 4.012 89 | 132      | 13.849 3 | 578      | 38.768 4 | 291      |
| 4.105 84 | 130      | 14.216 6 | 568      | 39.581 7 | 287      |
| 4.202 46 | 128      | 14.561 3 | 559      | 40.423 9 | 283      |
| 4.302 91 | 126      | 14.920 2 | 550      | 41.296 4 | 279      |



| $R^3$    | $1/3R^2$ | $R^3$   | $1/3R^2$ | $R^3$   | $1/3R^2$ | $R^3$   | $1/3R^2$ |
|----------|----------|---------|----------|---------|----------|---------|----------|
| 42.200 6 | (.0)275  | 106.887 | 148      | 216.324 | 925      | 489.054 | 537      |
| 43.138 4 | 271      | 107.980 | 147      | 219.880 | 915      | 495.965 | 532      |
| 44.111 5 | 267      | 109.091 | 146      | 223.535 | 905      | 503.040 | 527      |
| 44.865 4 | 264      | 110.222 | 145      | 227.292 | 895      | 510.285 | 522      |
| 45.641 2 | 261      | 111.372 | 144      | 231.155 | 885      | 517.705 | 517      |
| 46.439 6 | 258      | 112.542 | 143      | 235.129 | 875      | 525.307 | 512      |
| 47.261 5 | 255      | 113.733 | 142      | 238.803 | 866      | 533.097 | 507      |
| 48.108 0 | 252      | 114.944 | 141      | 242.575 | 857      | 541.081 | 502      |
| 48.980 0 | 249      | 116.179 | 140      | 246.447 | 848      | 547.614 | 498      |
| 49.878 7 | 246      | 117.434 | 139      | 250.423 | 839      | 554.278 | 494      |
| 50.805 2 | 243      | 118.713 | 138      | 254.507 | 830      | 561.078 | 490      |
| 51.760 8 | 240      | 120.015 | 137      | 258.704 | 821      | 568.020 | 486      |
| 52.746 8 | 237      | 121.342 | 136      | 263.017 | 812      | 575.106 | 482      |
| 53.764 3 | 234      | 122.692 | 135      | 266.952 | 804      | 582.340 | 478      |
| 54.815 1 | 231      | 124.068 | 134      | 270.987 | 796      | 589.726 | 474      |
| 55.900 5 | 228      | 125.470 | 133      | 275.124 | 788      | 597.271 | 470      |
| 57.022 2 | 225      | 126.898 | 132      | 279.367 | 780      | 604.978 | 466      |
| 58.182 0 | 222      | 128.354 | 131      | 283.721 | 772      | 612.851 | 462      |
| 59.381 6 | 219      | 129.838 | 130      | 288.189 | 764      | 620.897 | 458      |
| 60.623 0 | 216      | 131.351 | 129      | 292.776 | 756      | 629.122 | 454      |
| 61.908 2 | 213      | 132.893 | 128      | 296.890 | 749      | 637.528 | 450      |
| 63.239 6 | 210      | 134.466 | 127      | 301.101 | 742      | 646.124 | 446      |
| 64.153 9 | 208      | 136.070 | 126      | 305.412 | 735      | 654.914 | 442      |
| 65.090 4 | 206      | 137.706 | 125      | 309.828 | 728      | 663.907 | 438      |
| 66.050 0 | 204      | 139.375 | 124      | 314.351 | 721      | 673.106 | 434      |
| 67.033 4 | 202      | 141.078 | 123      | 318.985 | 714      | 682.520 | 430      |
| 68.041 4 | 200      | 142.816 | 122      | 323.734 | 707      | 692.155 | 426      |
| 69.074 9 | 198      | 144.590 | 121      | 328.602 | 700      | 702.020 | 422      |
| 70.134 8 | 196      | 146.401 | 120      | 333.594 | 693      | 709.573 | 419      |
| 71.222 2 | 194      | 148.250 | 119      | 338.713 | 686      | 717.263 | 416      |
| 72.338 0 | 192      | 150.139 | 118      | 343.964 | 679      | 725.092 | 413      |
| 73.483 1 | 190      | 152.068 | 117      | 349.352 | 672      | 733.064 | 410      |
| 74.658 9 | 188      | 154.039 | 116      | 354.883 | 665      | 741.184 | 407      |
| 75.866 3 | 186      | 156.053 | 115      | 359.741 | 659      | 749.455 | 404      |
| 77.106 6 | 184      | 158.111 | 114      | 364.710 | 653      | 757.882 | 401      |
| 78.381 0 | 182      | 160.214 | 113      | 369.795 | 647      | 766.467 | 398      |
| 79.691 0 | 180      | 162.365 | 112      | 375.000 | 641      | 775.215 | 395      |
| 81.037 9 | 178      | 164.564 | 111      | 380.327 | 635      | 784.131 | 392      |
| 82.423 1 | 176      | 166.813 | 110      | 385.782 | 629      | 793.220 | 389      |
| 83.848 3 | 174      | 169.113 | 109      | 391.369 | 623      | 802.485 | 386      |
| 85.315 0 | 172      | 171.467 | 108      | 397.091 | 617      | 811.932 | 383      |
| 86.825 0 | 170      | 173.877 | 107      | 402.955 | 611      | 821.566 | 380      |
| 88.380 1 | 168      | 176.343 | 106      | 408.964 | 605      | 831.392 | 377      |
| 89.982 1 | 166      | 178.868 | 105      | 415.124 | 599      | 841.416 | 374      |
| 91.633 1 | 164      | 181.454 | 104      | 421.440 | 593      | 851.642 | 371      |
| 93.335 2 | 162      | 184.103 | 103      | 427.918 | 587      | 862.078 | 368      |
| 95.090 7 | 160      | 186.817 | 102      | 433.445 | 582      | 872.727 | 365      |
| 96.901 9 | 158      | 189.599 | 101      | 439.091 | 577      | 883.599 | 362      |
| 98.771 4 | 156      | 192.450 | 100      | 444.861 | 572      | 894.698 | 359      |
| 100.702  | 154      | 195.669 | (.00)989 | 450.758 | 567      | 906.031 | 356      |
| 101.692  | 153      | 198.980 | 978      | 456.787 | 562      | 917.605 | 353      |
| 102.696  | 152      | 202.385 | 967      | 462.952 | 557      | 929.429 | 350      |
| 103.718  | 151      | 205.888 | 956      | 469.256 | 552      | 941.508 | 347      |
| 104.756  | 150      | 209.493 | 945      | 475.704 | 547      | 953.851 | 344      |
| 105.813  | 149      | 212.863 | 935      | 482.302 | 542      | 966.466 | 341      |
|          |          |         |          |         |          | 979.361 | 338      |
|          |          |         |          |         |          | 992.546 | 335      |



Fractional Powers, other than square and cube roots, fourth roots, etc., for practical purposes, cannot be found directly on a calculating machine. However, the Curta will be found very useful when used in conjunction with log tables.

For instance, if a number of quantities have to be raised to the same fractional power, the power can be set on SR. The log of each number in turn is looked up in the tables and turned into the CR. The antilog of the resultant PR is looked up in the tables.

Example:

|    |  |                |                  |
|----|--|----------------|------------------|
|    | 1.42125                                      |                |                  |
|    | 5.395  | Using Curta II |                  |
| C1 | /1.42125                                     | .7319914/      | C1.04034277725/* |
|    | Answer (from 7 figure log tables) = 10.97344 |                |                  |

The advantages of using a calculating machine are that the power can be set on the machine once for all similar computations and only the answer need be written down. The special advantage of the Curta is that it can be held over the log tables during the work facilitating the reading of the antilog and reducing the chances of error.

Any quadratic equation can be expressed in the form

$$x^2 + ax + b = 0,$$

the roots of which are:-

$$\frac{1}{2}(-a \pm (a^2 - 4b)^{\frac{1}{2}})$$

The square root of  $a^2 - 4b$  can, of course, be found on a Curta by one of the methods indicated in M 1 and the roots of the quadratic computed in this manner. If, however,  $a^2$  is very large in relation to  $4b$ , one of the roots will be the difference between two nearly equal quantities and too few significant figures may be given unless the square root of  $a^2 - 4b$  is computed to a very large number of figures.

In such cases, the root can be found very quickly to 6 significant figures on Curta I and 8 significant figures on Curta II merely by a process of division.

If the roots are real,  $-a$  is one approximate root and  $-b/a$  the other. Second approximations are given by  $-a + b/a$  and  $\frac{b}{-a + b/a}$

Example:

$$x^2 - 152x + 3 = 0 \quad \text{Using Curta II.}$$

One approximate root is  $3/152$

Another is  $152 - 3/152$

A closer approximation of the 1st root is  $3/(152 - 3/152)$  Using build up division but with Reversing Lever down to give complement of quotient.

|    |            |            |                        |
|----|------------|------------|------------------------|
| C7 | R /152     | 98026/ *   | /300048000 Clear dials |
| C8 | /151.98026 | 19739406 * | 300000005612556        |

One root is therefore .019739406

The other is 152, less the above root, i.e. 151.980260594



Using similar notation to that used for square and cube roots in M 1 and 2, any cubic equation can be reduced to the form

$$N^3 + aN = b \quad \dots\dots\dots (1)$$

If  $R$  is an approximate solution so that  $N = R + d$ , where  $d$  is small,  $(R + d)^3 + a(R + d) = b$ , and ignoring  $d^2 + d^3$

$$R^3 + 3R^2d + aN = b$$

Substituting  $N - R$  for  $d$ ,  $R^3 + 3R^2N - 3R^3 + aN = b$ ,

$$\text{and } N = \frac{b + 2R^3}{a + 3R^2} \quad \dots\dots\dots (2)$$

Thus, if an approximate root,  $R$ , has been found, a second order approximation can be found from equation (2) and thus any desired accuracy can be obtained by successive approximations.

Example:

Find the real roots of  $N^3 + 2N = 8 \quad \dots\dots\dots (3)$

An approximate root lies between 1.5 and 2, say 1.75.

Set 8 on PR, 2 on SR and multiply by -1.75 obtaining for  $N^3$ , 4.50

From a table of squares and cubes, e.g. Barlows', we find  $N^3$  should be 5.359375, so that 1.75 is too large. We therefore turn the handle positively until we get 4.660 on PR for 1.670 on CR for which  $N^3$ , i.e.  $1.67^3 = 4.657463$ . This is as near as we can get to 4 significant figures.

From equation (2) we have a second approximation

$$N = \frac{b + 2(1.670)^3}{2 + 3(1.670)^2} = \frac{8 + 2 \times 4.657463}{2 + 3 \times 2.7889} \quad (\text{from the tables})$$

$$= 1.6702447$$

Dividing  $N^3 + 2N - 8$  by  $N - 1.6702447$ , we get

$$N^2 + 1.6702447N + 4.7897174 = 0$$

the roots of which are imaginary.

So our only real root is 1.6702447.

Using a Curta II, the operations are as follows:-

|    |   |            |             |                     |                  |
|----|---|------------|-------------|---------------------|------------------|
| C8 |   | 0008/      | 1/          | 8/*                 | Clear CR         |
| C8 | R | 0002/      | 1.750/*     | 4.50                |                  |
|    |   |            | 1.670/*     | 4.66/               | Clear PR<br>& CR |
| C4 |   | 8/         | /1000       | 08/*                |                  |
|    |   | 4.657463/  | /3000       | 17.314926/*         |                  |
| C1 |   | /2.7889    | /3003       | 17.314926:008.3667* |                  |
|    |   | /2.0000    | /3004       | 17.314926:010.3667* | Clear CR         |
| C8 | R | /10.366700 | /1.6702447* | /2788767            |                  |

Answer = 1.6702447

In making the above calculation we have used each side of the PR to accumulate numerator and denominator of N. The CR figures are meaningless after the 2nd step until the last step as there is no necessity to clear CR.



SUMMATION OF SQUARES.

In many statistical and other problems it is necessary to sum the squares of a number of quantities,  $\sum x^2$ . Frequently it is also necessary to sum the quantities themselves,  $\sum x$ . Both summations can be performed simultaneously on the Curta, preferably on Model II, since on that machine the multiplier can be checked at each step of the operation.

We set the figure '1' on the left hand setting dial and  $x$  on the right of SR. Multiply by  $x$  so that  $x^2$  appears on the right side of PR and  $x$  both on the left of PR and on CR. We can either check and clear CR, accumulating  $\sum x$  on the left of PR, or check and clear the left side of PR (returning the clearing lever to the left side of PR) accumulating  $\sum x$  on CR. In either case  $\sum x^2$  is accumulated on the right of PR. Generally the second method is the best if a large number of quantities has to be squared and summed, since it is easier to check the multiplier on the left of PR, whilst CR provides a larger capacity for summing the quantities. On the other hand, when further operations have to be performed on  $\sum x^2$ , the former method may have advantages.

Example.

Find the standard deviation of the following quantities:-  
1239, 1347, 1196, 0985, 1082.

The standard deviation is given by:-  $S = \frac{\sqrt{\sum (x - \bar{x})^2}}{n}$ , where

$\bar{x}$  is the arithmetic mean of the quantities. This is more conveniently evaluated as  $S = \frac{1}{n} \sqrt{n \sum x^2 - (\sum x)^2}$ , so that we must find  $\sum x^2$  and

$\sum x$ . The steps on Curta II are as follows:-

|    |         |       |                 |          |
|----|---------|-------|-----------------|----------|
| C1 | 1//1239 | /1239 | 01239//1535121* | Clear CR |
| C1 | 1//1347 | /1347 | 02586//3349530* | Clear CR |
| C1 | 1//1196 | /1196 | 03782//4779946* | Clear CR |
| C1 | 1//0985 | /0985 | 04767//5750171* | Clear CR |
| C1 | 1//1082 | /1082 | 05849//6920895* | Clear CR |

We now multiply right side of PR by 5, by successive multiplication.

|    |    |          |                  |          |
|----|----|----------|------------------|----------|
| C7 | /4 | /6920895 | 05849//34604475* | Clear CR |
|----|----|----------|------------------|----------|

Then deduct the square of 5849 from right side of PR.



C4 R            1//5849            /5849            /393674\* Clear CR

Next find the square root of 393674 by method M 1.(5) using Table 101.

C1                /395554            /1                /789228\* Clear CR

C6                /794                /789228            /627.436260\* Clear CR

Now divide by 5

C8                /5                    125.487/\*        /1260

Note: Since the original observations included no decimal places, we cannot get greater accuracy in the standard deviation, unless of course the quantities were integers. We can thus take the standard deviation as 125.

Of course the number of squares to be summed will normally be many more than 5 but the principle is identical.

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ACCUMULATION OF PRODUCTS.

There are many problems in which it is necessary to sum the products  $\sum xy$  and  $\sum xz$  of two sets of quantities  $y$  and  $z$  multiplied by a third set of quantities  $x$ , which also has to be summed. All three sums can be evaluated simultaneously on the Curta. We set  $y$  on the left of SR,  $z$  on the right and multiply by  $x$ . If none of the dials are cleared, we obtain  $\sum xy$  on the left of PR,  $\sum xz$  on the right and  $\sum x$  on CR. It is desirable, of course, to check the latter with the sum of the  $x$ 's found by some other means, e.g. mentally or by adding-listing machine.

Example.

Evaluate  $E$  given by the following equation:-

$$E = \frac{30260 \sum (f_{\lambda} E_{\lambda}) + 9.531 \sum (\lambda E_{\lambda}) - 27047.861 \sum (E_{\lambda})}{8566.7}$$

where  $\lambda$ ,  $f_{\lambda}$ , and  $E_{\lambda}$  are given by the following table

| $\lambda$ | $f_{\lambda}$ | $E_{\lambda}$ |
|-----------|---------------|---------------|
| 300       | .553          | 1.34          |
| 305       | .669          | 1.54          |
| 310       | .811          | 1.76          |
| 315       | .893          | 2.03          |
| 320       | .936          | 2.16          |
| 325       | 1.000         | 2.26          |
| 330       | .982          | 2.22          |
| 335       | .888          | 2.03          |
| 340       | .790          | 1.80          |
| 345       | .667          | 1.57          |
| 350       | .517          | 1.34          |

Note: This example has been kindly supplied by the Physical Department of Beecham Research Laboratories Ltd. as a sample of the usefulness of the Curta Model II in accumulating two products and one multiplier simultaneously. The computation is used in the calculation of vitamin A content in percomorph liver oil, from ultra violet absorption spectra. Any readers who would like to examine the complete system of analysis and computation may obtain full details from London Automatic Business Machines Ltd., 15 Cromwell Road, London SW7.



The operations on the Curta II are:-

|    |            |         |                     |
|----|------------|---------|---------------------|
| C1 | 300//.553  | /1.34*  | 00402.00//0.74102*  |
| C1 | 305//.669  | /2.88*  | 00871.70//1.77128*  |
| C1 | 310//.811  | /4.64*  | 01417.30//3.19864*  |
| C1 | 315//.893  | /6.67*  | 02056.75//5.01143*  |
| C1 | 320//.936  | /8.83*  | 02747.95//7.03319*  |
| C1 | 326//1.000 | /11.09* | 03484.71//9.29319*  |
| C1 | 330//.982  | /13.31* | 04217.31//11.47323* |
| C1 | 335//.888  | /15.34* | 04897.36//13.27587* |
| C2 | 340//.790  | /17.14* | 05509.36//14.69787* |
| C1 | 345//.667  | /18.71* | 06051.01//15.74506* |
| C1 | 350//.517  | /20.05* | 06520.01//16.43784* |

Write down  $\Sigma(f_x E_x) = 16.43784$

$$\Sigma(N E_x) = 6520.01$$

$$\Sigma(E_x) = 20.05 \quad \text{and clear dials.}$$

|      |              |         |                |          |
|------|--------------|---------|----------------|----------|
| C8   | /16.43784    | 30260/  | 0497409.0384/  | Clear CR |
| C1   | /6520.01000  | /9.531  | 0559551.25371/ | Clear CR |
| C2 R | /27047.86100 | /20.050 | 0017241.64066/ | Clear CR |

Now we divide by 8566.7

|      |         |         |           |
|------|---------|---------|-----------|
| C8 R | /8566.7 | 2.0126/ | /30024000 |
|------|---------|---------|-----------|

Answer 2.013



A series  $a_0 + a_1x + a_2x^2 + \dots$

for any given value of  $x$  could be computed by raising  $x$  to the successive powers of 2, 3, etc., writing down the answers and accumulating the products of these answers and the appropriate coefficients  $a_2, a_3$ , etc. It is speedier and less liable to cause error, however, if we re-write the series in a form which will enable us to carry out the computation in a continuous operation, without writing down anything except the answer.

Suppose the powers in excess of 4 can be ignored, we may re-write the series:-

$$a_0 + x(a_1 + x(a_2 + x(a_3 + x.a_4)))$$

Thus, we can set  $a_4$  on SR, multiply by  $x$ ; add  $a_3$ , multiply by  $x$ ; add  $a_2$ , multiply by  $x$ ; add  $a_1$ , multiply by  $x$  and add  $a_0$ . We then obtain the required value on PR.

Example: With a Curta Model II

Evaluate  $1 + 5x + 9x^2 + 13x^3 + 17x^4 + 21x^5 + \dots$

to 5 places of decimals where  $x = .0925$

We can ignore the terms containing powers of  $x$  greater than 6.

|    |           |            |                    |          |
|----|-----------|------------|--------------------|----------|
| C3 | /21       | /.0925     | /1.9425 *          | Clear CR |
| C5 | /17       | /1.0000    | /18.9425 *         | Clear CR |
| C6 | /.0924    | /18.9425   | /1.75218125 *      | Clear CR |
| C8 | /13.0     | 1/         | /14.75218125 *     | Clear CR |
| C8 | /.092400  | /14.752181 | /1.364576742525 *  | Clear CR |
| C8 | /9.00000  | 1/         | /10.364576742525 * |          |
| C8 | /10.36458 | 0          | 9/6742525 *        | Clear PR |
| C1 |           | /925       | /.958723650 *      | Clear CR |
| C8 | /5.00     | 1/         | /5.958723650 *     | Clear CR |
| C8 | /.092400  | 5.9587236  | /.5511819330050 *  |          |

Answer = 1.55118



Convergent series of form  $\frac{a_1}{a_2} + \frac{a_1}{a_2} \frac{b_1}{b_2} + \frac{a_1}{a_2} \frac{b_1}{b_2} \frac{c_1}{c_2}$  etc. can be evaluated on Curta II in a continuous operation.

We split SR and PR, and set  $a_1$  on the left of SR and  $a_2$  on the right. If we build up to 1 on the left of PR, we shall obtain  $\frac{a_1}{a_2}$  on the right. We may either clear the left side of PR or build up to 1 to the left of the 15th dial of PR, i.e. off the register. So we now have 0 on the left and the 1st term of the series on the right of PR.

We change the settings to  $b_1$  and  $b_2$  and build up the left hand side of PR to the amount which showed on the right hand side, i.e.  $\frac{a_1}{a_2}$ . Thus we add  $\frac{a_1}{a_2} \times \frac{b_1}{b_2}$  to the right hand side. We now have the 1st term on the left and the sum of the first two terms on the right.

We proceed in this way until the left and right hand sides of PR agree to the required number of significant figures, obtaining the sum of  $n$  terms on the left and  $n + 1$  terms on the right.

In the following example we shall omit CR, which is not used in this type of calculation, and show the 2 sides of SR and PR separately.



Example:

Evaluate the following series to six places of decimals.

$$.3 + \frac{.3 \times .5}{2!} + \frac{.3 \times .5 \times .7}{3!} + \dots$$

Split SR and PR between the 8th and 9th dials.

|    | <u>SR</u> |          | <u>PR</u> |           |
|----|-----------|----------|-----------|-----------|
|    | <u>L</u>  | <u>R</u> | <u>L</u>  | <u>R</u>  |
| C8 | /1        | /3       | .0/       | .3/       |
| C7 | /2        | /5       | .3/       | .375/     |
| C6 | /3        | /7       | .375/     | .3925/    |
| C5 | /4        | /9       | .3925/    | .3964375/ |
| C5 | /5        | /11      | .3964375  | .39730375 |
| C4 | /6        | /13      | .3973039  | .39749147 |
| C3 | /7        | /15      | .3974915  | .39753167 |
| C2 | /8        | /17      | .3975315  | .39754017 |
| C2 | /9        | /19      | .3975405  | .39754207 |
| C1 | /10       | /21      | .3975415  | .39754228 |
| C1 | /11       | /23      | .3975426  | .39754251 |

Answer = .397542

If we have a polynomial  $a_0x^n + a_1x^{n-1} + \dots$ , then the  $n^{\text{th}}$  differences are constant and equal to  $a_0n(\delta x)^2$ , where  $\delta x$  is the interval of  $x$ . To construct the polynomial for successive values of  $x$  we require 'n' consecutive values of the polynomial and the next value as a numerical check. We calculate the successive differences in these values and then build up from the constant  $n^{\text{th}}$  difference.

Example:

$$y = x^4 + 3x^3 + 7x^2 + 5x + 2 \text{ from } 0 \text{ with intervals of } .1$$

The  $4^{\text{th}}$  difference is  $24 \times (.1)^4 = .0024$

Taking as the 1st five terms  $-.2, -.1, 0, .1$  and  $.2$ , for ease of calculation we get:-

| x   | y      | $\delta y$ | $\delta^2 y$ | $\delta^3 y$ | $\delta^4 y$ |
|-----|--------|------------|--------------|--------------|--------------|
| -.2 | 1.2576 | .3095      |              |              |              |
| -.1 | 1.5671 | .4329      | .1234        |              |              |
| 0   | 2.0000 | .5731      | .1402        | .0168        | .0024        |
| .1  | 2.5731 | .7325      | .1594        | .0192        |              |
| .2  | 3.3056 |            |              |              |              |

We shall use the symbols  $y_1, \delta y_{1\frac{1}{2}}, \delta^2 y_{\frac{1}{2}}$ , etc. to represent respectively the value of  $y$  corresponding to  $x = .1$ , the value of the second difference between  $\delta y_1$  and  $\delta y_2$ , the value of the third difference between  $\delta^2 y_0$  and  $\delta^2 y_1$ , etc.

We require a Curta Model II.

Set  $\delta^2 y_1$  on left of SR and  $y_1$  on the right.

Turn the handle once.

Set  $\delta^3 y_{\frac{1}{2}}$  on left of SR and  $\delta y_{1\frac{1}{2}}$  on the right.

Add .0024 to left hand side of SR and turn handle once. Check that right side of PR reads  $y_2$

From now on, after each turn of the handle, increase left side of PR by 24 and add to right side the amount showing in the left of PR. To avoid mistakes, it will be as well to move the carriage to position 8 so that right side of SR comes under left side of PR. Alter the setting dials and return the carriage to position 1 before making the next turn, thus:-



|    |                         |      |                   |
|----|-------------------------|------|-------------------|
| C1 | .1594,,002.5731         | /.1  | /.1594,,002.5731* |
|    | $\delta^3 y$ $\delta y$ | x    | $\delta^2 y$ y    |
| C1 | .0216,,000.7325         | /.2  | /.1810,,003.3056* |
| C8 | .0240,,000.9135         |      |                   |
| C1 |                         | /.3  | /.2050,,004.2191* |
| C8 | .0264,,001.1185         |      |                   |
| C1 |                         | /.4  | /.2314,,005.3376* |
| C8 | .0288,,001.3499         |      |                   |
| C1 |                         | /.5  | /.2602,,006.6875* |
| C8 | .0312,,001.6101         |      |                   |
| C1 |                         | /.6  | /.2914,,008.2976* |
| C8 | .0336,,001.9015         |      |                   |
| C1 |                         | /.7  | /.3250,,010.1991* |
| C8 | .0360,,002.2265         |      |                   |
| C1 |                         | /.8  | /.3610,,012.4256* |
| C8 | .0384,,002.5875         |      |                   |
| C1 |                         | /.9  | /.3994,,015.0131* |
| C8 | .0408,,002.9869         |      |                   |
| C1 |                         | /1.0 | /.4402,,018.0000* |

The above table gives the values of y and the successive differences, in this order,  $\delta^3 y$ ,  $\delta y$ ,  $\delta^2 y$  and y.

The final value of y should be checked. This will prove that the calculation has been made correctly, but not that the values of y have been written down correctly. This can be proved by differencing the last column and checking the first differences with the second column of figures.

Owing to the compactness of the Curta, it will be found a simple matter to add or subtract figures from the setting dials. For polynomials of degree higher than 4, the calculation will have to be performed in 2 or more parts; for instance, with a polynomial of order 6, we can find the 2nd difference, as above, and build up the functions themselves from the second differences.

If a large volume of such work has to be done, it is best to use a National 3000 Class Accounting Machine, or, if the polynomial is of order greater than 6, a Logabax Accounting



## I N T E R P O L A T I O N

Interpolation is the process of finding the value of a function corresponding to a value of the argument between two values for which the value of the function is given.

There are many methods of interpolation, some of which are of little practical value and others which are appropriate for differing circumstances.

Here we confine ourselves to indicating the Curta technique applicable to the five methods most commonly useful.

1. Linear Interpolation
2. Neville's method.
3. Lagrange's Formula.
4. Everett's Formula.
5. Bessel's Formula.

Of these, linear interpolation is used wherever it gives sufficient accuracy. In other cases, Bessels is the most useful but requires tables of Bessels coefficients and the computation of successive differences to the order necessary to give the required accuracy. Everett's method is useful where differences of order more than four are necessary, since only even order differences are involved.

Lagrange's method also uses a table of coefficients but requires no differencing and Neville's method requires no table of coefficients and gives an automatic check on the accuracy of the computation but is rather more labourous than the last three.

### Notation.

Various notations are used in textbooks and tables, and our reason for using that given below is that it appears to us to be short, unambiguous and comparatively easy to type on an ordinary typewriter.

If we have a function  $y = f(x)$  and require the value  $f_n$  corresponding to  $x_n = x_0 + nSx$  where  $Sx$  is the interval in the argument, the central difference table is



| Argument | $f(x)$   | Differences              |                 |                           |              |
|----------|----------|--------------------------|-----------------|---------------------------|--------------|
|          |          | 1st                      | 2nd             | 3rd                       | 4th          |
| $x_{-2}$ | $f_{-2}$ |                          |                 |                           |              |
| $x_{-1}$ | $f_{-1}$ | $\delta_{-1\frac{1}{2}}$ | $\delta_{-1}^2$ |                           |              |
| $x_0$    | $f_0$    | $\delta_{-\frac{1}{2}}$  | $\delta_0^2$    | $\delta_{-\frac{1}{2}}^3$ | $\delta_0^4$ |
| $x_1$    | $f_1$    | $\delta_{\frac{1}{2}}$   | $\delta_1^2$    | $\delta_{\frac{1}{2}}^3$  |              |
| $x_2$    | $f_2$    | $\delta_{1\frac{1}{2}}$  |                 |                           |              |

Examples.

In the examples given here for each method we shall use the same set of figures from a table of  $\sin x$  for  $x$  given in radians at intervals of 0.1. In this way the reader will be able to compare the time taken and accuracy of the different methods.

| $x$          | $\sin x$ | $\delta$  | $\delta^2$ | $\delta^3$ | $\delta^4$ | $\delta^5$ |
|--------------|----------|-----------|------------|------------|------------|------------|
| $x_{-2}$ 4.0 | -0.75680 |           |            |            |            |            |
|              |          | $-.06148$ |            |            |            |            |
| $x_{-1}$ 4.1 | .81828   |           | $+0.00818$ |            |            |            |
|              |          | $.05330$  |            | $+0.00053$ |            |            |
| $x_0$ 4.2    | .87158   |           | 871        |            | $-0.00008$ |            |
|              |          | $.04459$  |            | 45         |            | $-0.00003$ |
| $x_1$ 4.3    | .91617   |           | 916        |            | 11         |            |
|              |          | $.03543$  |            | 34         |            |            |
| $x_2$ 4.4    | .95160   |           | 950        |            |            |            |
|              |          | $.02593$  |            |            |            |            |
| $x_3$ 4.5    | .97753   |           |            |            |            |            |

If the maximum contribution to an interpolate from the lowest order of differences is not to exceed .5 in the last significant figure, the limiting values for the differences ignored are given in 'Interpolation and Allied Tables' (H.M. Stationery Office) as follows:-



| Difference neglected. | Limiting value. |
|-----------------------|-----------------|
| 2nd                   | 4               |
| 3                     | 60              |
| 4                     | 20              |
| 5                     | 500             |
| 6                     | 100             |
| 7                     | 3500            |
| 8                     | 400             |
| 9                     | 20,000          |
| 10                    | 2,000           |

Thus, in the above example we can ignore differences over the 2nd. If we ignore 2nd differences we can only rely on the first two decimal places in the interpolate, though the third decimal place is not likely to be inaccurate by more than about 1 unit in the 3rd decimal place.

### 1. Linear Interpolation.

The simplest form of interpolation is 'linear'. This assumes that a function  $y = f(x)$  can be represented between points  $x_0$  and  $x_1$  by a straight line through these points, i.e.  $f(x)$  can be represented by a polynomial of degree 1. We will call this polynomial

$$f_{0,1} = \frac{f_0(x_1 - x_n) + f_1(x_n - x_0)}{x_1 - x_0}$$

For unit interval in the argument, this reduces to

$$\begin{aligned} f_{0,1} &= f_0 + (x_n - x_0)(f_1 - f_0) \\ &= f_0 + n\delta_{\frac{1}{2}} \end{aligned}$$

#### Example (i).

Find the value of  $\sin 4.215$  radians by linear interpolation.

There is no object in using more than 3 figures in  $\sin x$  as we can only rely on two decimal places in the answer.

We could set  $-f_0$ , i.e. .872, and multiply by  $\frac{x_1 - x_n}{x_1 - x_0}$  that is .85

Then set  $-f_1$ , i.e. .916, and multiply by  $\frac{x_n - x}{x_1 - x_0}$  that is .15

obtaining .87860 in the product dials, so that the answer is .88 to two places of decimals. Actually the 1st 3 figures .879 are, in this case, correct.



In order to check the settings, however, it is generally considered preferable to set  $f_0$  and turn into PR. Then set  $S_n$  and turn into PR, checking that we have  $f_1 = f_0 + S_n$  in PR. If we now make one negative turn to restore  $f_0$  in PR and then turn through .15 we set .87860 on PR, as before, thus:-

|    |        |      |                    |
|----|--------|------|--------------------|
| C3 | / .872 | /100 | / .87200*          |
|    | / .044 | /200 | / .91600* Check PR |
|    |        | /100 | / .87200*          |
| C2 |        | /115 | / .87860*          |

Answer .879

Note: If we were to set the full 5 figure values we should get

|    |          |      |                      |
|----|----------|------|----------------------|
| C3 | / .87158 | /100 | / .8715800*          |
|    | / .04459 | /200 | / .9161700* Check PR |
|    |          | /100 | / .8715800           |
| C2 |          | /115 | / .8782685*          |

As the correct 5 figure interpolate is .87883, we are not gaining in accuracy by working to more than 3 figures.

## 2. Neville's Method.

If linear interpolation gives insufficient accuracy, we may represent the function by a polynomial of higher degree. Both Aitken's and Neville's methods are based on using a polynomial of degree sufficient to give the required accuracy but Neville's method is slightly preferable for use with the Curta.

Suppose the 1st degree polynomial between points  $x_0$  and  $x_1$  is represented by  $f_{0,1} = \frac{-f_1(x_1 - x_n) + f_0(x_0 - x_n)}{x_1 - x_0}$

We can obtain a second degree polynomial, represented by  $f_{-1,0,1}$ , between  $x_{-1}$ ,  $x_0$  and  $x_1$  formed in the same way from  $f_{0,-1}$  and  $f_{0,1}$ , thus:-

$$f_{-1,0,1} = \frac{f_{0,-1}(x_1 - x_n) - f_{0,1}(x_{-1} - x_n)}{x_1 - x_{-1}}$$



We can set out the scheme of computation thus for a 4th degree polynomial:-

|          |                |          |             |               |                 |
|----------|----------------|----------|-------------|---------------|-----------------|
| $x_{-2}$ | $x_{-2} - x_n$ | $f_{-2}$ |             |               |                 |
|          |                |          | $f_{-2,-1}$ |               |                 |
| $x_{-1}$ | $x_{-1} - x_n$ | $f_{-1}$ |             | $f_{-2,-1,0}$ |                 |
|          |                |          | $f_{-1,0}$  |               | $f_{-2,-1,0,1}$ |
| $x_0$    | $x_0 - x_n$    | $f_0$    |             | $f_{-1,0,1}$  |                 |
|          |                |          | $f_{0,1}$   |               | $f_{-1,0,1,2}$  |
| $x_1$    | $x_1 - x_n$    | $f_1$    |             | $f_{0,1,2}$   |                 |
|          |                |          | $f_{1,2}$   |               |                 |
| $x_2$    | $x_2 - x_n$    | $f_2$    |             |               |                 |

Example (ii).

Find the value of  $\sin 4.215$  radians, from the table already given, by Neville's method.

The only figures which it is necessary to write down are:-

| $x$ | $x - x_n$ | 1st degree | 2nd degree | 3rd degree |
|-----|-----------|------------|------------|------------|
| 4.0 | -215      | -0.888982  |            |            |
| 4.1 | -115      | 79575      | -0.878869  | -0.878836  |
| 4.2 | -015      | 78268      | 23         | 4          |
| 4.3 | +085      | 86054      | 52         |            |
| 4.4 | +185      |            |            |            |

The polynomials have been computed to 6 figures but we can only get 5 figure accuracy in the answer which may thus be taken as -0.87883.



The operations on the Curta are as follows:-

|     |    |          |       |           |               |
|-----|----|----------|-------|-----------|---------------|
| (a) | C1 | / .75680 | 9/885 | 9/1296800 |               |
|     | C1 | / .81828 | /100  | /8889820* | Clear CR & PR |
|     | C1 |          | 9/85  | 9/8772580 |               |
|     | C1 | / .87158 | /100  | /8795750* | - do -        |
|     | C1 |          | /85   | /7408430  |               |
|     | C1 | / .91617 | /100  | /8782685* | - do -        |
|     | C1 |          | /185  | /16949145 |               |
|     | C1 | / .95160 | /100  | /8860545* |               |

Note that since the interval between the x's is .1 the figure in PR can be divided mentally by  $x_{-1}$   $x_{-2}$  etc. to obtain  $f_{-2}$ ,  $-1$ , etc.

The CR is only cleared after each pair of multiplications, when it should read 100. Note also that it is only necessary to set each value of  $\sin x$  once.

|     |    |       |      |           |                                |
|-----|----|-------|------|-----------|--------------------------------|
| (b) | C1 | 88982 | 9/85 | 9/8665270 |                                |
|     | C1 | 79575 | /200 | /15773895 | Divide by 2 &<br>Clear CR & PR |
|     | C1 |       | /85  | /6763875  |                                |
|     | C1 | 78268 | /200 | /15764695 | - do -                         |
|     |    |       | /185 | /14479580 |                                |
|     | C1 | 86054 | /200 | /15770390 | - do -                         |

Note that it is unnecessary to set the first 8' which is common to all the values of the 1st degree polynomial.

|     |    |    |      |        |                                |
|-----|----|----|------|--------|--------------------------------|
| (c) | C1 | 69 | /85  | /5865  |                                |
|     | C1 | 23 | /300 | /10810 | Divide by 3 &<br>Clear CR & PR |
|     | C1 |    | /185 | /4255  |                                |
|     | C1 | 52 | /300 | /10235 | - do -                         |

It is only necessary to set the last 2 figures of the 2nd degree polynomial.

Since the two values of the 3rd degree polynomial agree to 5 decimal places, it is unnecessary to proceed further.



### 3. Lagrange's Formula.

If we extend the value of the four point polynomial  $f_{-1,0,1,2}$  in terms of  $f_{-1}$ ,  $f_0$ ,  $f_1$ , and  $f_2$ , we have an expression

$$f_n = f_{-1} L_{-1}(n) + f_0 L_0(n) + f_1 L_1(n) + f_2 L_2(n)$$

where  $L_{-1}$ ,  $L_0$ ,  $L_1$  and  $L_2$  are 3rd degree polynomials in  $n$ , and are known as Lagrange Interpolation Coefficients. They can be evaluated and tabulated for various values of  $n$  and for various degrees of the polynomial, see Chambers' 6 figure tables which give the coefficients for 4 and 6 point interpolation. A table for four point interpolation and two figure values for  $n$  is given in Curta Table No.103.

The value of  $f_n$  can be found by accumulating the products  $f_{-1} L_{-1}(n)$  etc.

#### Example (iii).

Find the value of  $\sin 4.215$  radians by Lagrange's Formula.

For  $n = .15$  and a 4-point polynomial the Lagrange coefficients (see Curta Table No.103) are:-

|          |         |
|----------|---------|
| $L_{-1}$ | -.03931 |
| $L_0$    | +.90419 |
| $L_1$    | +.15956 |
| $L_2$    | -.02444 |

All we need to do is to set the values of  $f_{-1}$  etc in turn, multiply by the coefficients and accumulate the products thus:-

|    |        |         |              |
|----|--------|---------|--------------|
| C1 | .81828 | 9/6069  | 9/678334132* |
| C1 | .87158 | /.86488 | /7559073334* |
| C1 | .91617 | 1.02444 | /9020914186* |
| C1 | .95160 | 1.00000 | /8788343146* |

Answer -0.87883

Note that it is not necessary to clear CR as the sum of the Lagrange coefficients should be 1.0 and the multipliers can be checked in this way.



4. Everett's Formula.

We can express Lagrange's Formula for a 4-point polynomial in terms of  $f_0$  and 1st and 2nd differences, by taking

$$f_{-1} = f_0 - S_{\frac{1}{2}} + S_0^2, \quad f_1 = f_0 + S_{\frac{1}{2}}, \quad f_2 = f_0 + 2S_{\frac{1}{2}} + S_1^2.$$

Lagrange's Formula then becomes

$$f_n = f_0 L + S_{\frac{1}{2}}(2L_2 + L_1 - L_{-1}) + L_{-1}S_0^2 + L_2S_1^2$$

$$\text{Now } L = 1 \text{ and } 2L_2 + L_1 - L_{-1} = n$$

$$\text{So } f_n = f_0 + n S_{\frac{1}{2}} + L_{-1}S_0^2 + L_2S_1^2$$

This is Everett's formula for a 4-point polynomial.  $L_{-1}$  and  $L_2$  are usually written  $E_0^{ii}$  and  $E_1^{ii}$  respectively. The full formula is  $f_n = f_0 + n S_{\frac{1}{2}} + E_0^{ii} S_0^2 + E_1^{ii} S_1^2 + E_0^{iv} S_0^4 + E_1^{iv} S_1^4 + \text{etc.}$ , and it will be noticed that all differences after the first are even-order.

Everett's interpolation coefficients are tabulated in 'Interpolation and Allied Tables'. The 'throwback' can be used as with Bessel's Formula.

Curta Table No.103 gives values of  $E_0^{ii}$  and  $E_1^{ii}$  for 2 figure values of  $n$ .

Example iv.

Find the value of  $\sin 4.215$  radians, by Everett's Formula.

Again we can ignore differences greater than the 2nd.

$$E_0^{ii} = -.03931 \quad E_1^{ii} = -.02444 \quad \text{from Curta Table No.103.}$$

|    |        |         |               |          |
|----|--------|---------|---------------|----------|
| C6 | .87158 | 1.00000 | /.8715800000* | Clear CR |
| C5 | .04459 | /.15000 | /.8782685000* | Clear CR |
| C4 | .00871 | /.03931 | /.8786108901* | Clear CR |
| C4 | .00916 | /.02444 | /.8788347605  | Clear CR |

Answer -0.87883



### 5. Bessel's Formula.

This formula expresses the interpolated value of  $f$  in terms of central differences, thus:-

$$f_n = f_0 + n S_{\frac{1}{2}} + B^{ii}(S_0^2 + S_1^2) + B^{iii} S_{\frac{1}{2}}^3 + B^{iv}(S_0^4 + S_1^4) + \text{etc.}$$

where  $B^{ii}$ ,  $B^{iii}$ ,  $B^{iv}$ , etc. are known as Bessels Interpolation

Coefficients dependent on the value of  $n$ , and are tabulated, as for instance in 'Interpolation and Allied Tables'. A table of  $B^{ii}$  and  $B^{iii}$  to 3 decimal places is given in Curta Table No.104.

A good approximation is given by applying Bessel's formula correct to 2nd or 3rd differences only and modifying  $S^2$  by subtracting  $.1848^4$ . This is known as 'throwback' from 4th to 2nd differences and may be used if the 4th difference does not exceed 1000.

#### Example (v).

Find the value of  $\sin 4.215$  radians by Bessel's Formula.

In this case we can ignore differences greater than the 2nd, as already stated. Therefore all we have to do is to look up the value of  $B^{ii}$  for  $n$  in the tables, e.g. Curta Table No.104 gives  $-.03187$  for  $B^{ii}$ . We set  $f_0$  and transfer to PR, set  $S_{\frac{1}{2}}^1$  and multiply by  $n$ , and set  $S_0^2 + S^2$  and multiply by  $.032$ , thus:-

|    |        |         |               |          |
|----|--------|---------|---------------|----------|
| C6 | .87158 | 1.00000 | /.8715800000* | Clear CR |
| C5 | .04459 | /.15000 | /.8782685000* | Clear CR |
| C4 | .01787 | /.03187 | /.8788380169* |          |

Answer  $-.87884$

Note that  $S_0^4 + S_1^4 = .00019$  so that the 'throwback' from 4th to 2nd difference could only amount to  $.00019 \times .184 \times .03187 = .0000011141752$



| n    | $B^{ii}$<br>-0.0 | $B^{iii}$<br>+0.0- | n    | n    | $B^{ii}$<br>-0.0 | $B^{iii}$<br>+0.0- | n    |
|------|------------------|--------------------|------|------|------------------|--------------------|------|
| 0.00 | 0000             | 0000               | 1.00 | 0.25 | 4687             | 0781               | 0.75 |
| .01  | 0247             | 0081               | .99  | 6    | 4810             | 0770               | .74  |
| .02  | 0490             | 0157               | .98  | 7    | 4927             | 0756               | .73  |
| .03  | 0727             | 0228               | .97  | 8    | 5040             | 0739               | .72  |
| .04  | 0960             | 0294               | .96  | 9    | 5147             | 0721               | .71  |
| 0.05 | 1187             | 0356               | 0.95 | 0.30 | 5250             | 0700               | 0.70 |
| .06  | 1410             | 0414               | .94  | 1    | 5347             | 0677               | .69  |
| .07  | 1627             | 0467               | .93  | 2    | 5440             | 0653               | .68  |
| .08  | 1840             | 0515               | .92  | 3    | 5527             | 0626               | .67  |
| .09  | 2047             | 0560               | .91  | 4    | 5610             | 0598               | .66  |
| 0.10 | 2250             | 0600               | 0.90 | 0.35 | 5687             | 0569               | 0.65 |
| .11  | 2447             | 0636               | .89  | 6    | 5760             | 0538               | .64  |
| .12  | 2640             | 0669               | .88  | 7    | 5827             | 0505               | .63  |
| .13  | 2827             | 0697               | .87  | 8    | 5890             | 0471               | .62  |
| .14  | 3010             | 0722               | .86  | 9    | 5947             | 0436               | .61  |
| 0.15 | 3187             | 0744               | 0.85 | 0.40 | 6000             | 0400               | 0.60 |
| .16  | 3360             | 0762               | .84  | 1    | 6047             | 0363               | .59  |
| .17  | 3527             | 0776               | .83  | 2    | 6090             | 0325               | .58  |
| .18  | 3690             | 0787               | .82  | 3    | 6127             | 0286               | .57  |
| .19  | 3847             | 0795               | .81  | 4    | 6160             | 0246               | .56  |
| 0.20 | 4000             | 0800               | 0.80 | 0.45 | 6187             | 0206               | 0.55 |
| .21  | 4147             | 0802               | .79  | 6    | 6210             | 0166               | .54  |
| .22  | 4290             | 0801               | .78  | 7    | 6227             | 0125               | .53  |
| .23  | 4427             | 0797               | .77  | 8    | 6240             | 0083               | .52  |
| .24  | 4560             | 0790               | .76  | 9    | 6247             | 0042               | .51  |
| 0.25 | 4687             | 0781               | 0.75 | 0.50 | 6250             | 0000               |      |

Examples.  $n = 0.15$      $B^{ii} = -0.03187$      $B^{iii} = +0.00744$   
 $n = 0.55$      $B^{ii} = -0.06187$      $B^{iii} = -0.00206$



CURTA TABLE No. 103. LAGRANGE & EVERETT INTERPOLATION COEFFICIENTS.

| n    | $L_{-1} = E_0^{ii}$ | $L_0$   | $L_1$   | $L_2 = E_1^{ii}$ | n    |
|------|---------------------|---------|---------|------------------|------|
|      | -                   | +       | +       | -                |      |
| 0.00 | 0.00000             | 1.00000 | 0.00000 | 0.00000          | 1.00 |
| .01  | .00328              | .99490  | .01005  | .00167           | 0.99 |
| .02  | .00647              | .98960  | .02020  | .00333           | .98  |
| .03  | .00955              | .98411  | .03044  | .00500           | .97  |
| .04  | .01254              | .97843  | .04077  | .00666           | .96  |
| 0.05 | 0.01544             | 0.97256 | 0.05119 | 0.00831          | 0.95 |
| .06  | .01824              | .96651  | .06169  | .00996           | .94  |
| .07  | .02094              | .96027  | .07228  | .01161           | .93  |
| .08  | .02355              | .95386  | .08294  | .01325           | .92  |
| .09  | .02607              | .94726  | .09369  | .01488           | .91  |
| 0.10 | 0.02850             | 0.94050 | 0.10450 | 0.01650          | 0.90 |
| .11  | .03084              | .93357  | .11538  | .01811           | .89  |
| .12  | .03309              | .92646  | .12634  | .01971           | .88  |
| .13  | .03525              | .91920  | .13735  | .02130           | .87  |
| .14  | .03732              | .91177  | .14843  | .02288           | .86  |
| 0.15 | 0.03931             | 0.90419 | 0.15956 | 0.02444          | 0.85 |
| .16  | .04122              | .89645  | .17075  | .02598           | .84  |
| .17  | .04304              | .88856  | .18199  | .02751           | .83  |
| .18  | .04477              | .88052  | .19328  | .02903           | .82  |
| .19  | .04643              | .87233  | .20462  | .03052           | .81  |
| 0.20 | 0.04800             | 0.86400 | 0.21600 | 0.03200          | 0.80 |
| .21  | .04949              | .85553  | .22742  | .03346           | .79  |
| .22  | .05091              | .84692  | .23888  | .03489           | .78  |
| .23  | .05224              | .83818  | .25037  | .03631           | .77  |
| .24  | .05350              | .82931  | .26189  | .03770           | .76  |
| 0.25 | 0.05469             | 0.82031 | 0.27344 | 0.03906          | 0.75 |



CURTA TABLE No. 103.2. LAGRANGE & EVERETT INTERPOLATION COEFFICIENTS.

| n    | $L_{-1} = E_0^{ii}$ | $L_0$   | $L_1$   | $L_2 = E_1^{ii}$ | n    |
|------|---------------------|---------|---------|------------------|------|
|      | -                   | +       | +       | -                |      |
| 0.25 | 0.05469             | 0.82031 | 0.27344 | 0.03906          | 0.75 |
| .26  | .05580              | .81119  | .28501  | .04040           | .74  |
| .27  | .05683              | .80194  | .29661  | .04172           | .73  |
| .28  | .05779              | .79258  | .30822  | .04301           | .72  |
| .29  | .05868              | .78309  | .31986  | .04427           | .71  |
| 0.30 | 0.05950             | 0.77350 | 0.33150 | 0.04550          | 0.70 |
| .31  | .06025              | .76380  | .34315  | .04670           | .69  |
| .32  | .06093              | .75398  | .35482  | .04787           | .68  |
| .33  | .06154              | .74407  | .36648  | .04901           | .67  |
| .34  | .06208              | .73405  | .37815  | .05012           | .66  |
| 0.35 | 0.06256             | 0.72394 | 0.38981 | 0.05119          | 0.65 |
| .36  | .06298              | .71373  | .40147  | .05222           | .64  |
| .37  | .06333              | .70343  | .41312  | .05322           | .63  |
| .38  | .06361              | .69304  | .42476  | .05419           | .62  |
| .39  | .06384              | .68256  | .43639  | .05511           | .61  |
| 0.40 | 0.06400             | 0.67200 | 0.44800 | 0.05600          | 0.60 |
| .41  | .06410              | .66136  | .45959  | .05685           | .59  |
| .42  | .06415              | .65064  | .47116  | .05765           | .58  |
| .43  | .06413              | .63985  | .48270  | .05842           | .57  |
| .44  | .06406              | .62899  | .49421  | .05914           | .56  |
| 0.45 | 0.06394             | 0.61806 | 0.50569 | 0.05981          | 0.55 |
| .46  | .06376              | .60707  | .51713  | .06044           | .54  |
| .47  | .06352              | .59601  | .52854  | .06103           | .53  |
| .48  | .06323              | .58490  | .53990  | .06157           | .52  |
| .49  | .06289              | .57372  | .55123  | .06206           | .51  |
| 0.50 | 0.06250             | 0.56250 | 0.56250 | 0.06250          | 0.50 |

Examples.

$$n = 0.35$$

$$L_0 = + .06256$$

$$n = 0.75$$

$$E_0^{ii} = - 0.05469$$



COTTON CLOTH CALCULATIONS

Calculations of warp and weft weight in lbs involve division by the 'Hank Length' and the 'Counts' of yarn and weft. This may be carried out on the Curta by the method of transfer multiplication given in A.3 but, since Hank Length is a constant, 840, a little time will be saved by using a table of reciprocals of Hank Length x Counts - Curta Table 20.

(a) Warp weight in lbs =

$$\frac{\text{No. of ends in width} \times \text{Tape length in yards}}{\text{Hank length} \times \text{Counts of yarn}}$$

Example (1)

|             |           |
|-------------|-----------|
| No. of ends | 21424     |
| Tape Length | 106 yards |
| Counts      | 38        |

From Table 20 the reciprocal of 38 x 840 is .00003133, so all we have to do is to evaluate 21424 x 106 x .00003133 by successive multiplication (A.1), setting eight decimal places.

```
C1   /3133   /106   /332098   Clear CR
C6   /21423  /332098 /7114867552
```

Answer 71.15 lbs. or 71 lbs. 2 oz.



(b) Weft weight in lbs =

$$\frac{\text{Cloth Length in yds.} \times \text{P.P. inches} \times \text{width in Reed}}{\text{Hank Length} \times \text{Counts of weft}}$$

Example (2)

|                |          |
|----------------|----------|
| Cloth Length   | 124 yds. |
| P.P. inches    | 64       |
| Width in Reed  | 36       |
| Counts of weft | 34       |

From Table 20 the reciprocal of 34 x 840 is .00003501

Evaluating 124 x 64 x 36 x 3501 and resetting eight decimal places, we get by successive multiplication 10.002

|    |       |         |              |          |
|----|-------|---------|--------------|----------|
| C1 | /124  | /64     | /7936        | Clear CR |
| C4 | /35   | /7936   | /285696      |          |
| C6 | /3500 | /285696 | /10.00221696 |          |

Answer 10 lb.



COTTON CLOTH CALCULATIONS

IRTA TABLE No. 20

RECIPROCAL OF COUNTS x HANK LENGTH

| <u>Count</u> | <u>Reciprocal</u> | <u>Count</u> | <u>Reciprocal</u> | <u>Count</u> | <u>Reciprocal</u> | <u>Count</u> | <u>Reciprocal</u> |
|--------------|-------------------|--------------|-------------------|--------------|-------------------|--------------|-------------------|
| 4            | 29762             | 30           | 3968              | 62           | 1920              | 94           | 1266              |
| 5            | 23810             | 32           | 3720              | 64           | 1860              | 96           | 1240              |
| 6            | 19841             | 34           | 3501              | 66           | 1804              | 98           | 1215              |
| 7            | 17007             | 36           | 3307              | 68           | 1751              | 100          | 1190              |
| 8            | 14881             | 38           | 3133              | 70           | 1701              | 102          | 1167              |
| 9            | 13228             | 40           | 2976              | 72           | 1653              | 104          | 1145              |
| 10           | 11905             | 42           | 2834              | 74           | 1609              | 106          | 1123              |
| 12           | 9921              | 44           | 2706              | 76           | 1566              | 108          | 1102              |
| 14           | 8503              | 46           | 2588              | 78           | 1526              | 110          | 1082              |
| 16           | 7440              | 48           | 2480              | 80           | 1488              | 112          | 1063              |
| 18           | 6614              | 50           | 2381              | 82           | 1452              | 114          | 1044              |
| 20           | 5952              | 52           | 2289              | 84           | 1417              | 116          | 1026              |
| 22           | 5411              | 54           | 2205              | 86           | 1384              | 118          | 1009              |
| 24           | 4960              | 56           | 2126              | 88           | 1353              | 120          | 992               |
| 26           | 4579              | 58           | 2053              | 90           | 1323              | 122          | 976               |
| 28           | 4252              | 60           | 1984              | 92           | 1294              | 124          | 960               |

The figures given above are the last figures of eight decimal places





